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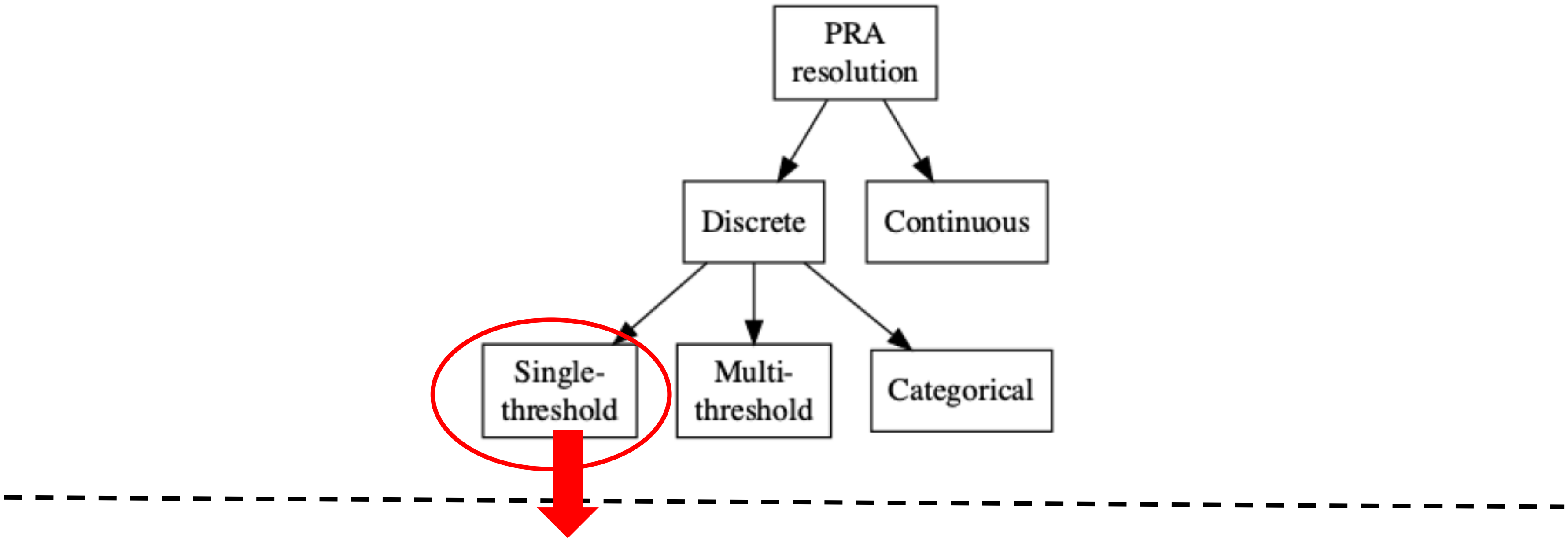
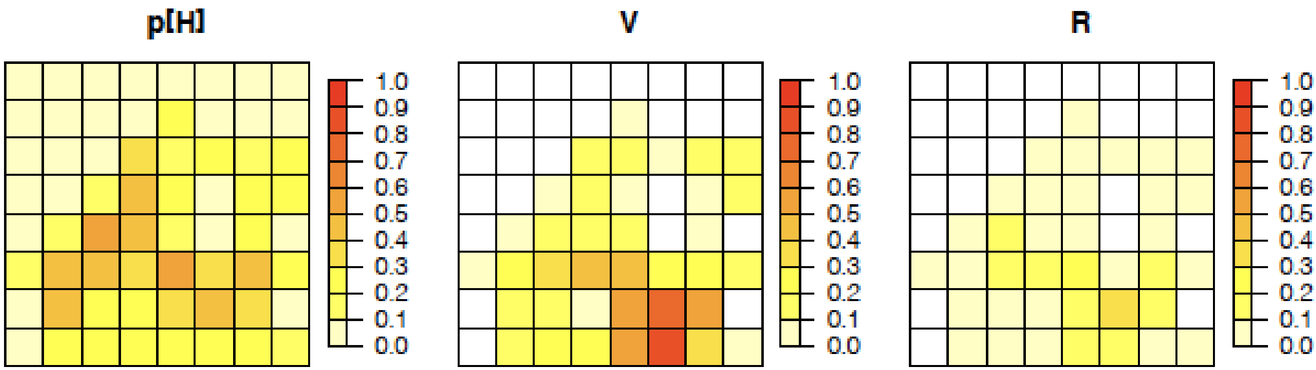
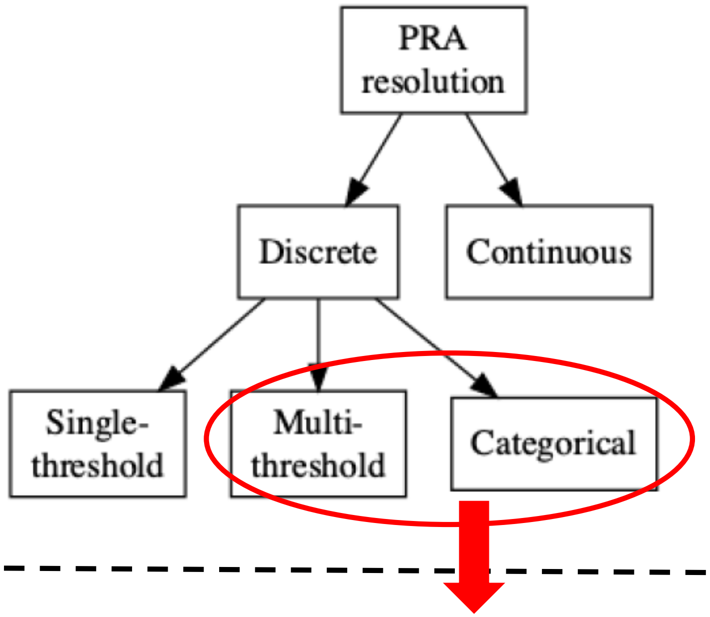
3.
Beyond the basic theory

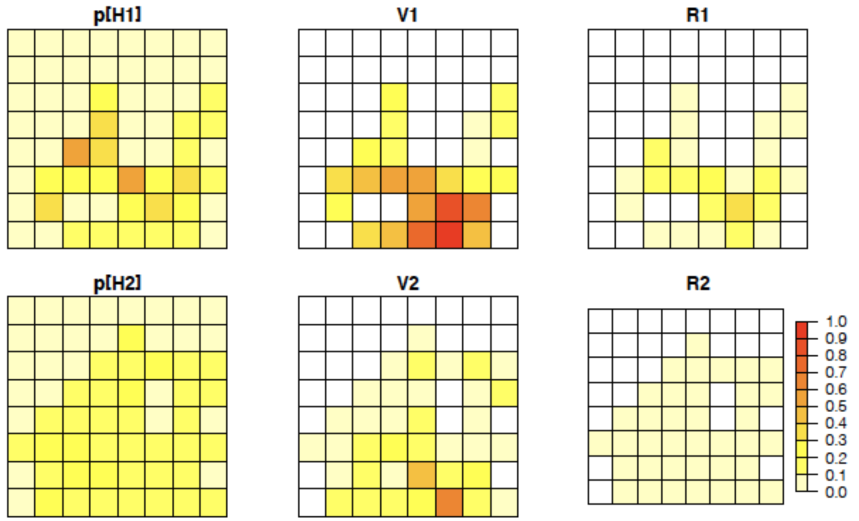
Extending the PRA to 2 hazardous conditions or more





Extending the PRA to 2 hazardous conditions or more





Drought level (or category) 1

Drought level (or category) 2

Multi-threshold PRA

$$p[H_i] = p[thr_{i-1} \leq x < thr_i],$$
$$V_i = E[z|x \geq thr_n] - E[z|thr_{i-1} \leq x < thr_i],$$
$$R_i = p[H_i]\,V_i,$$

where $i = 1, .., n$ and $thr_0 = -\infty$

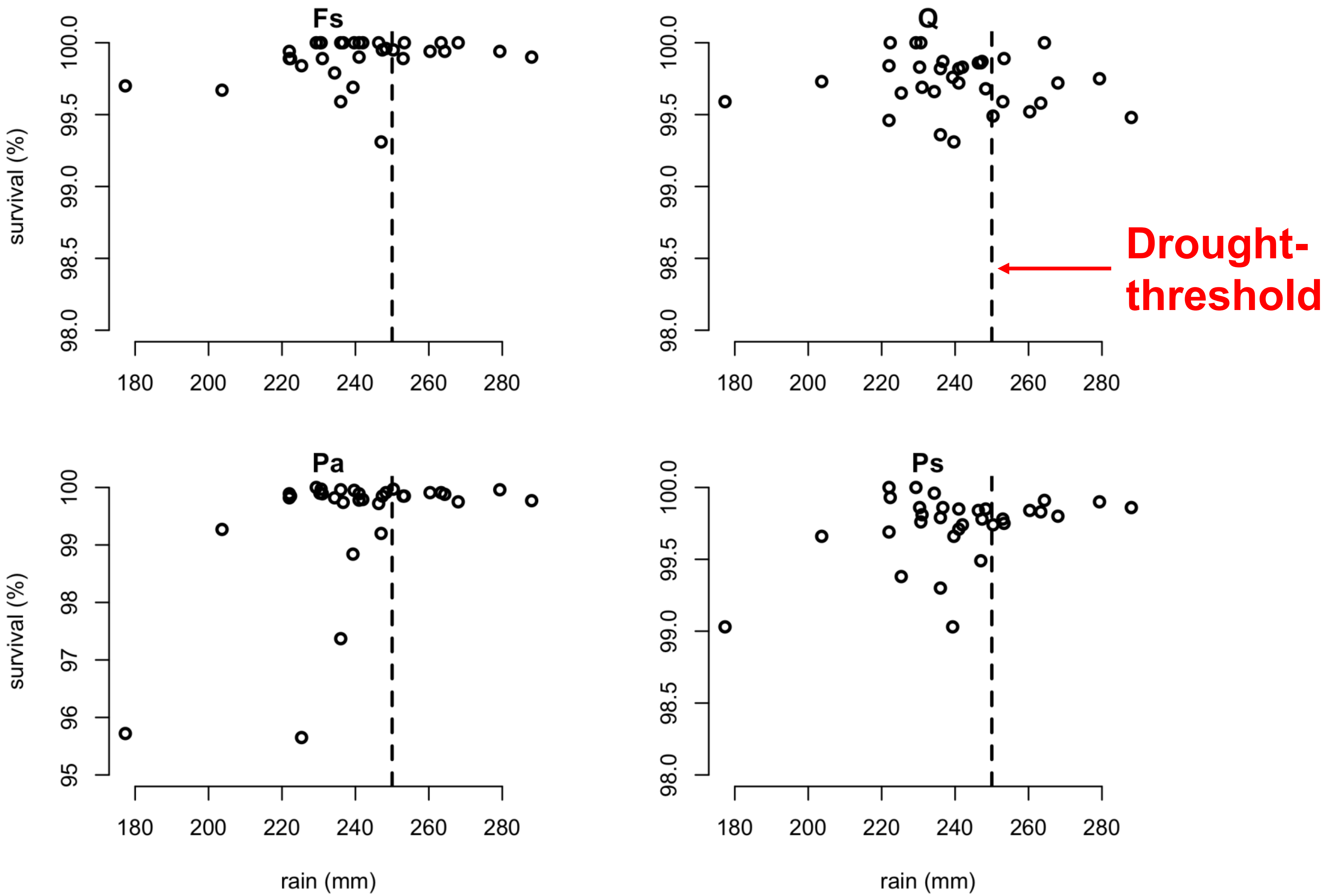
Retrieving total (single-threshold) PRA

$$p[H] = \sum p[H_i],$$
$$V = \sum \frac{p[H_i]}{p[H]} V_i,$$
$$R = \sum R_i,$$
$$= p[H]\,V.$$

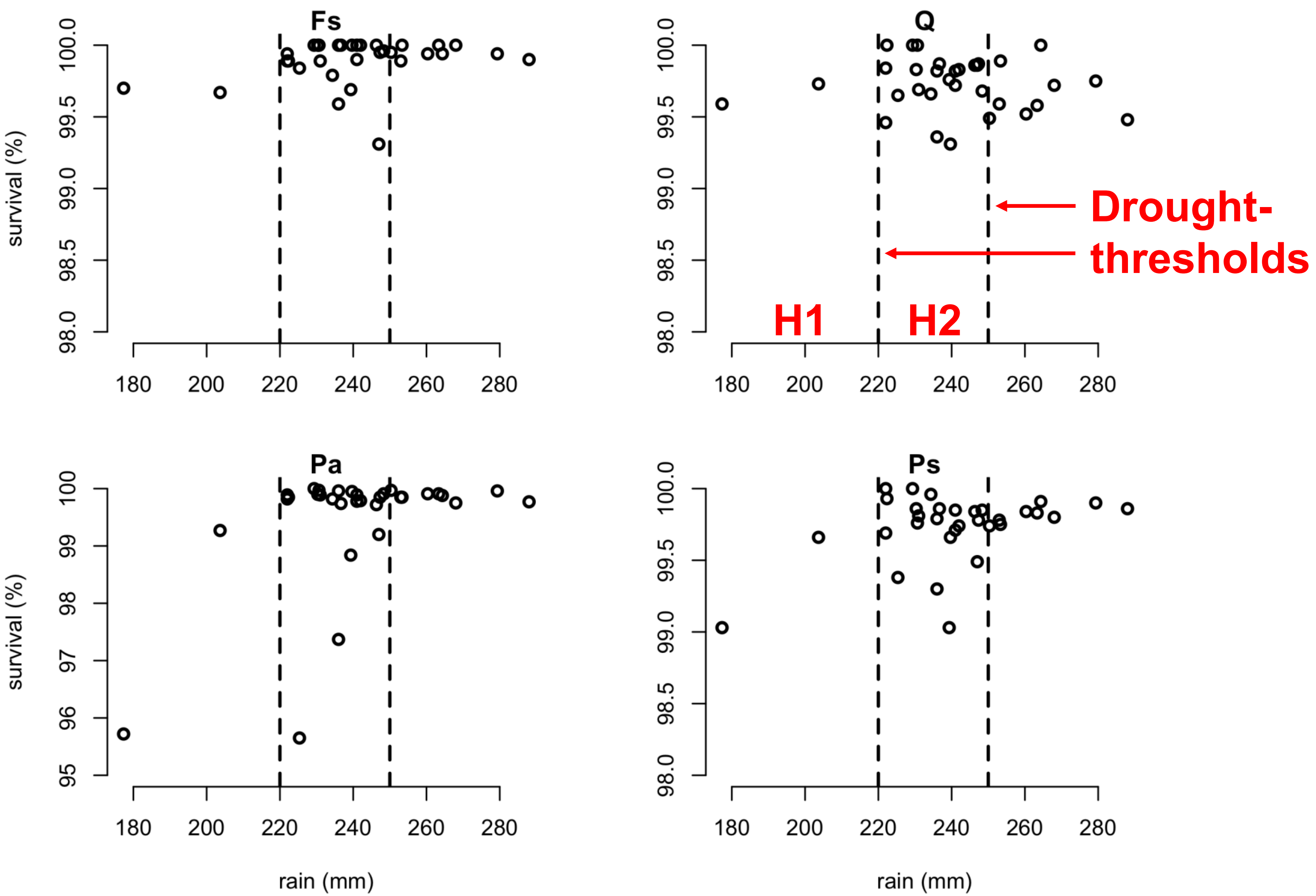
‘PRAm’: R-function for multi-threshold PRA

```
PRAm <- function( x, z, thr=-1:1 ) {
  n          <- length(x) ; n_thr <- length(thr)
  n_H        <- pH <- V <- R <- s_pH <- s_V <- s_R <- rep(NA,n_thr)
  H          <- vector("list",n_thr)
  H[[1]]     <- which( x < thr[1] ) ; n_H[1] <- length(H[[1]])
  for(i in 2:n_thr) { H[[i]] <- which( thr[i-1] <= x & x < thr[i])
                      n_H[i] <- length(H[[i]]) }
  n_notH     <- n - sum(n_H) ; H.all <- which( x < thr[n_thr] )

  pH         <- n_H / n      ; s_pH  <- sqrt( pH*(1-pH) / n )
  Ez_notH    <- mean( z[-H.all] )
  s_Ez_notH <- sqrt( var(z[-H.all] ) / n_notH )
  for(i in 1:n_thr) { Ez_Hi    <- mean( z[ H[[i]]])
                      s_Ez_Hi <- sqrt( var(z[ H[[i]]]) /  n_H[i] )
                      V[i]     <- Ez_notH - Ez_Hi
                      s_V[i]   <- sqrt( s_Ez_notH^2 + s_Ez_Hi^2 ) }
  R          <- pH * V
  s_R        <- sqrt( s_pH^2 * s_V^2 + s_pH^2 * V^2 + pH^2 * s_V^2 )

  R.sum <- sum(R) ; pH.sum <- sum(pH) ; V.wsum <- R.sum / pH.sum
  return( list( sum = c( pH.sum=pH.sum, V.wsum=V.wsum, R.sum=R.sum ),
                seq = cbind( thr, pH, V, R, s_pH, s_V, s_R ) ) )
}
```

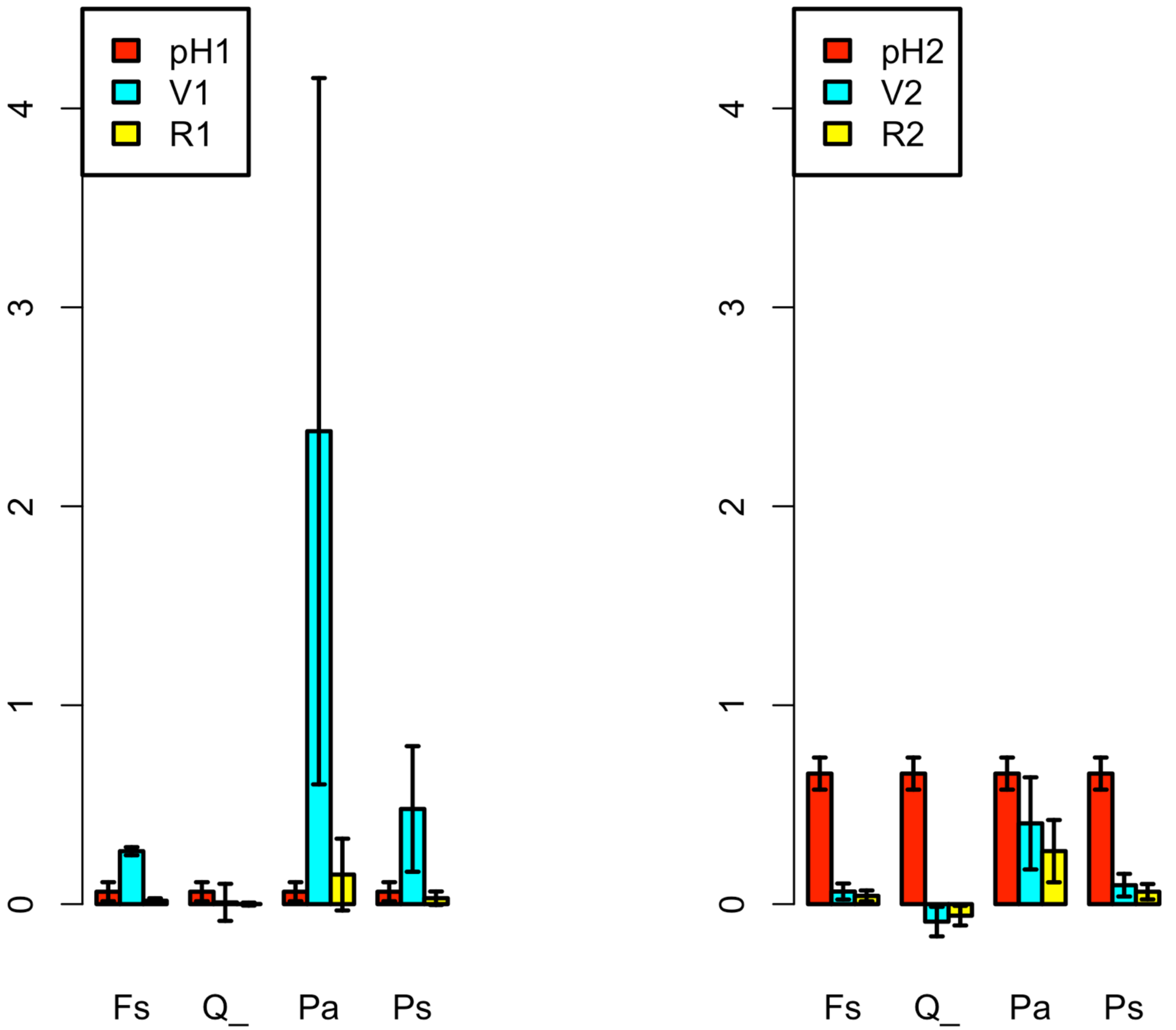
Forest survival data from Germany



Forest survival data from Germany



Forest data from Germany: Multi-threshold PRA



Forest data from Germany: EXERCISE 2

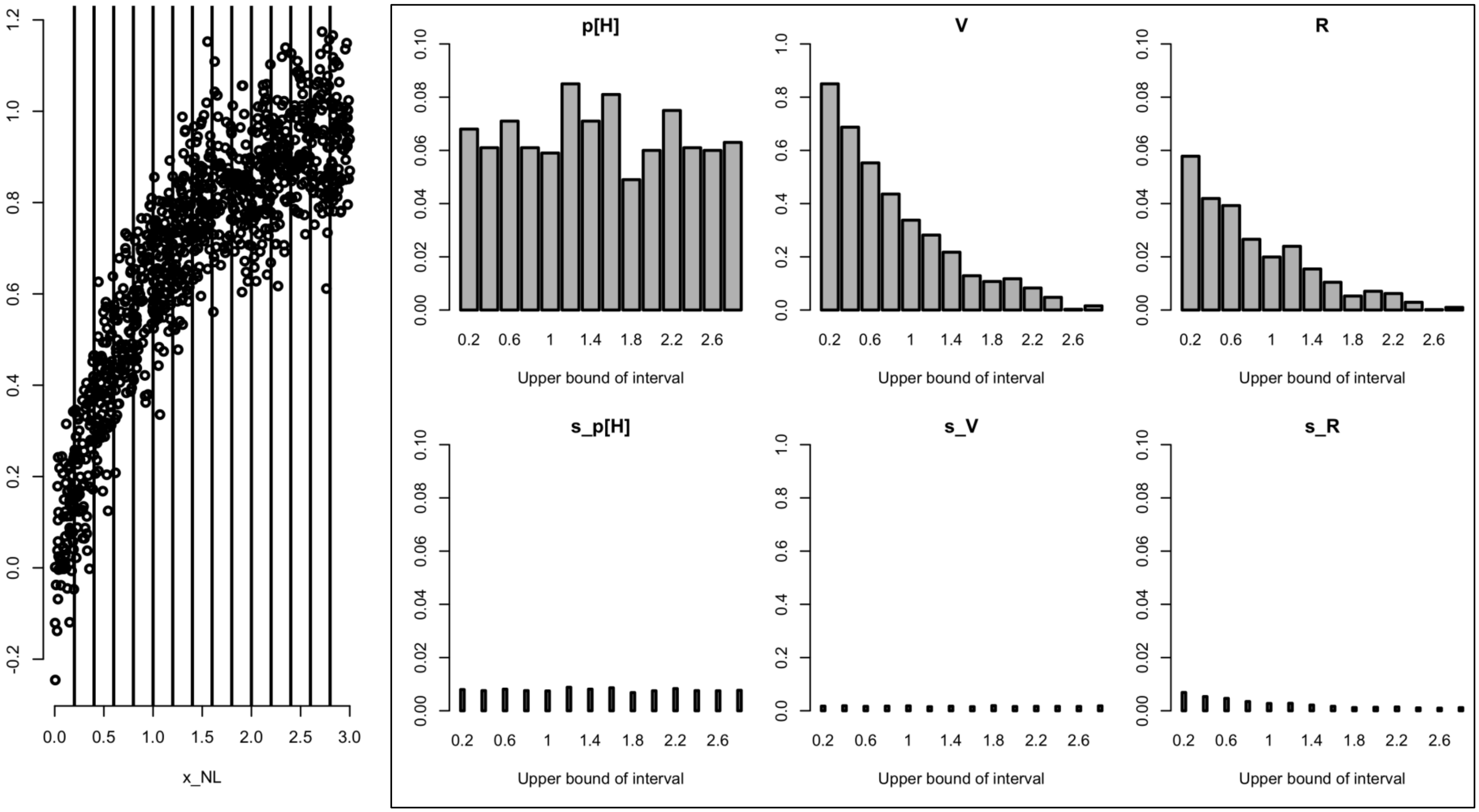
1. Change code: choose a series of multiple thresholds.
2. Discuss limitations of the multiple-threshold-PRA.

Forest data from Germany: EXERCISE 2

1. Change code: choose a series of multiple thresholds.

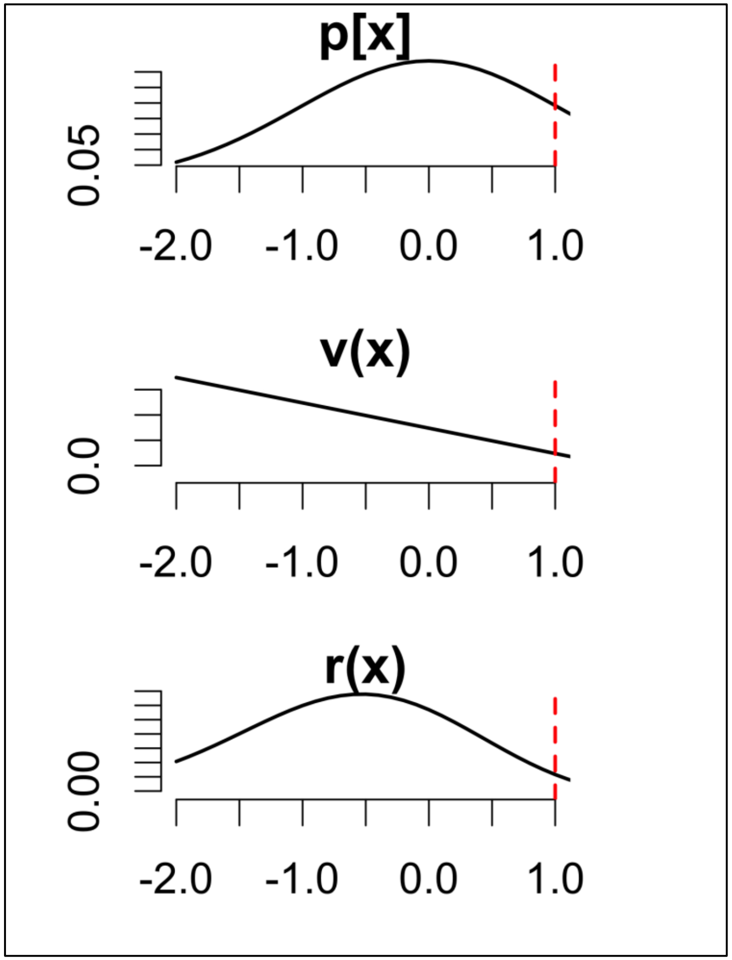
2. Discuss limitations of the multiple-threshold-PRA.
   Possible answers:
   - Mostly the same as for the single-threshold PRA.
   - Low n even more critical here?
   - …

Multi-threshold PRA on a rich dataset



Continuous single-threshold PRA: Bivariate Gaussian

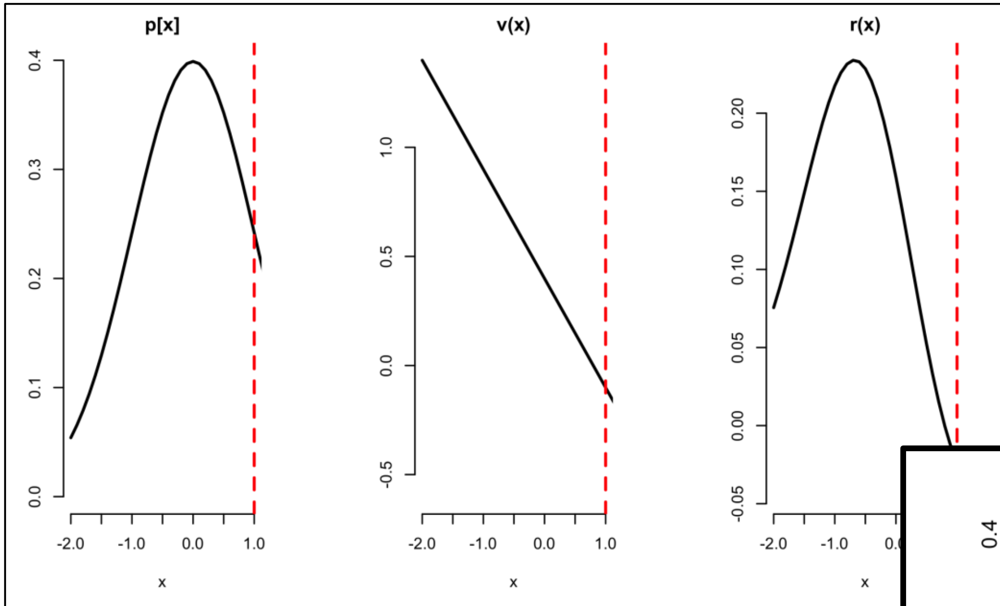
$$v(x) = E[z|x \geq thr] - E[z|x],$$
$$r(x) = p[x]\, v(x).$$

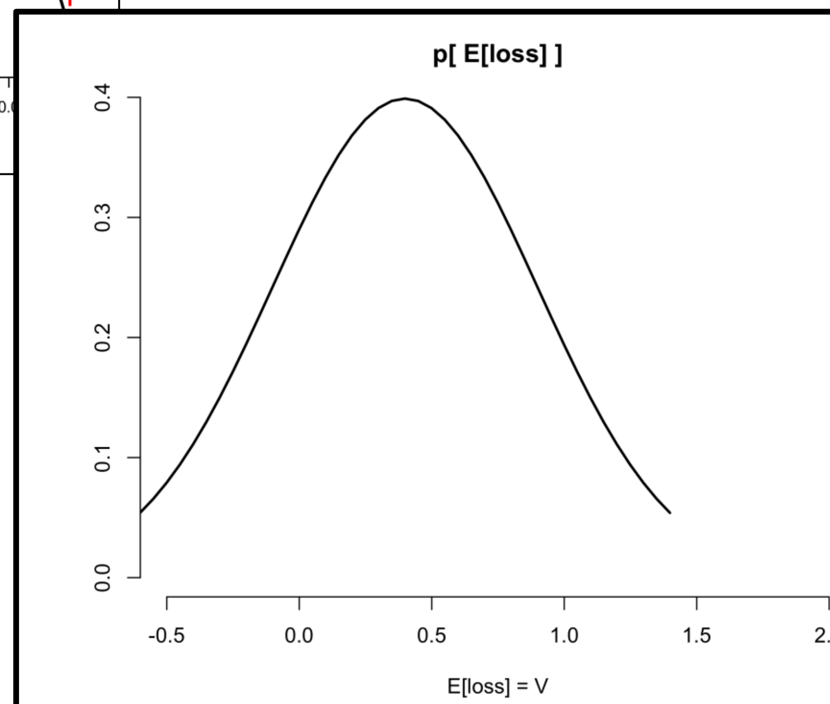


$$p[H] = \int_{x=-\infty}^{thr} p(x)dx,$$
$$V = \int_{x=-\infty}^{thr} \frac{p[x]}{p[H]} v(x)dx,$$
$$R = \int_{x=-\infty}^{thr} r(x)dx,$$
$$= p[H]\, V.$$

Retrieving total (single-threshold) PRA

Loss distribution: Useful or not?



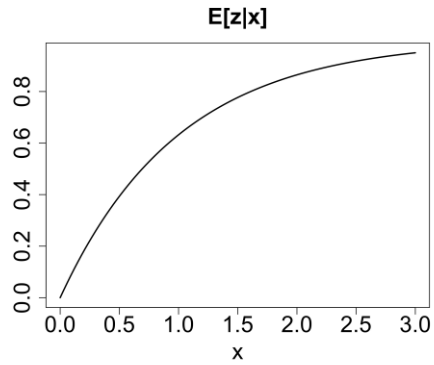


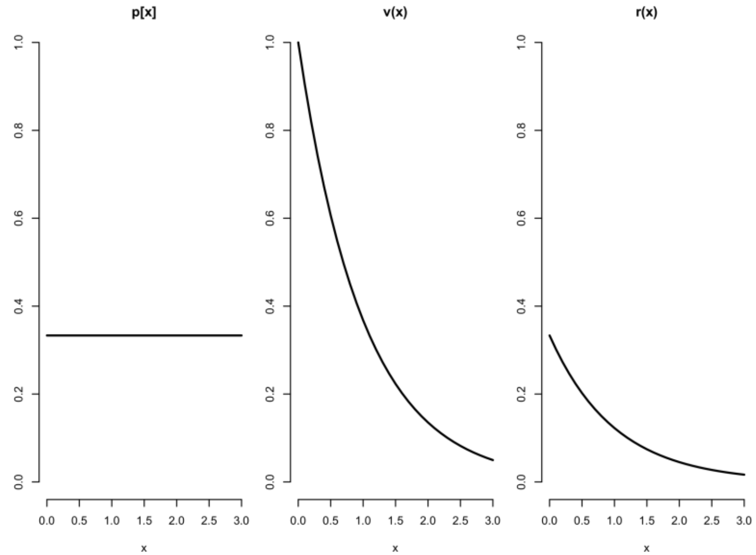
Continuous zero-threshold PRA: nonlinear model

$$E[z|x] = 1 - e^{-x}$$
$$z_{max} = 1$$
$$x \sim U[0,3]$$

$$p[x] = \frac{1}{3}; \quad x \in [0,3]$$
$$v(x) = z_{max} - E[z|x]$$
$$= e^{-x}$$
$$r(x) = p[x]v(x)$$
$$= \frac{1}{3}e^{-x}$$

$$R = \int_0^3 r(x)dx$$
$$= \frac{1}{3}(1 - e^{-3})$$
$$\approx 0.32$$



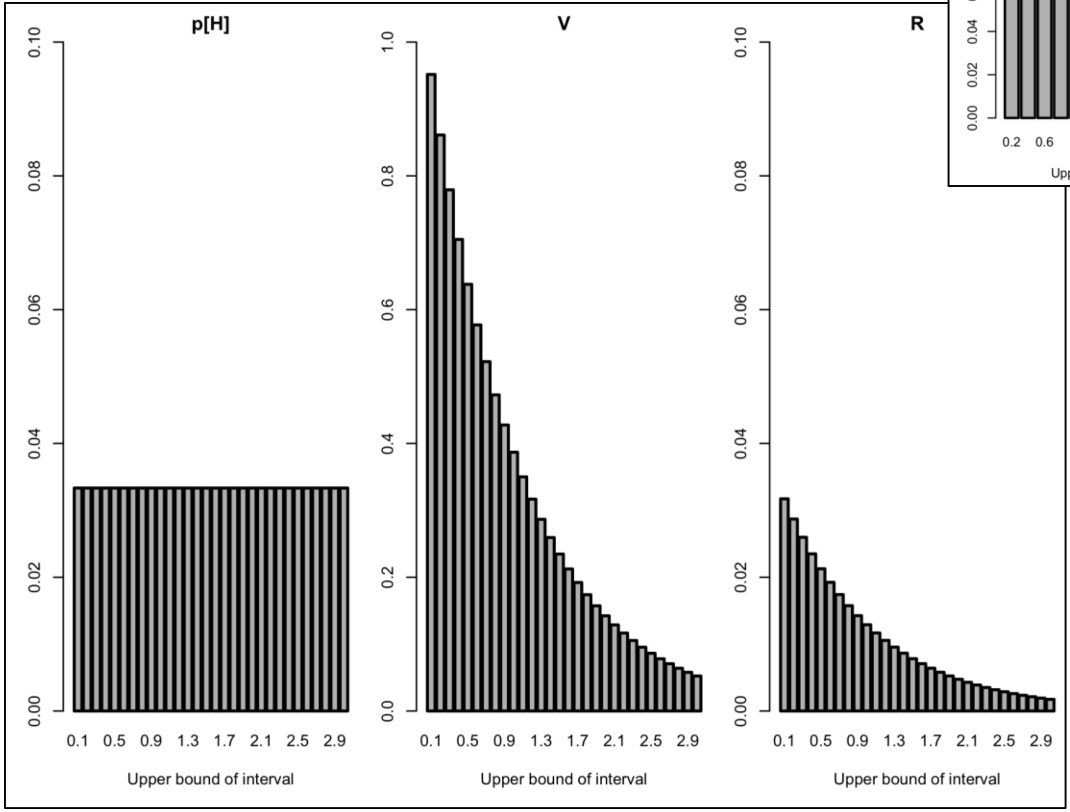


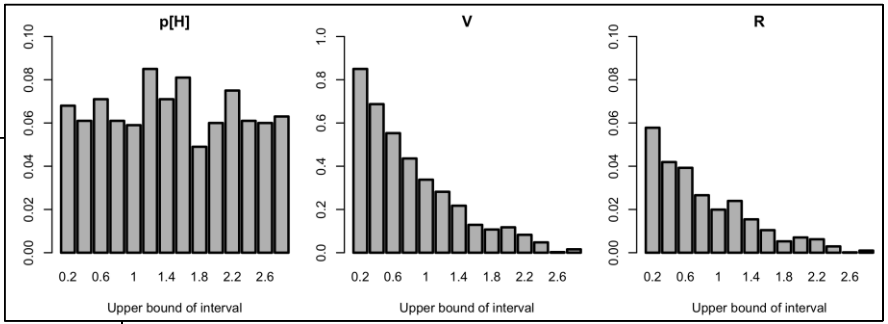
Discretizing a continuous PRA

$$R_a^b = \int_a^b r(x)dx = \frac{1}{3}(e^{-a} - e^{-b})$$

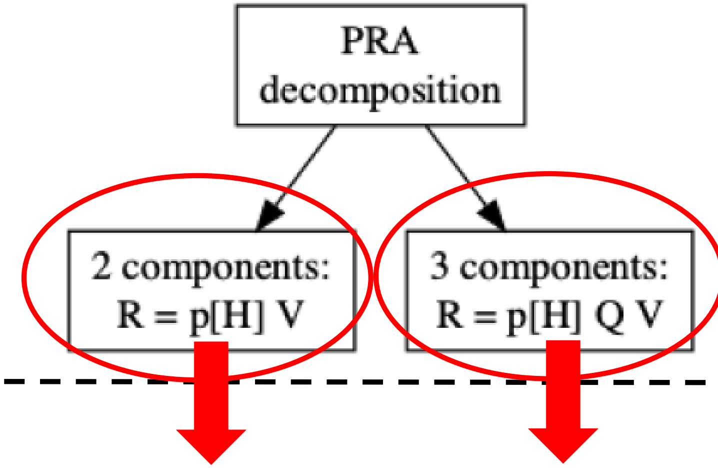
$$pH_a^b = \int_a^b p(x)dx = \frac{1}{3}(b - a)$$

$$V_a^b = R_a^b \,/\, pH_a^b$$

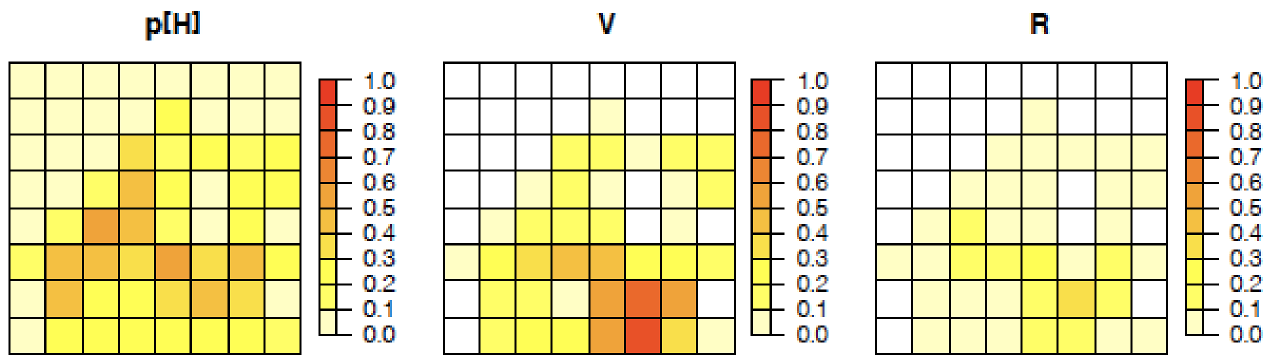




Extending the PRA from 2 risk-components to 3



64 different
2-component
PRAs at cell level:



1 single
3-component PRA
at regional level:

P[H] = 0.27	Q = 37/64	V = 0.26	R = 0.04

Extension to R = Q p[H] V

$g\ m^{-2}\ y^{-1}$			-	$g\ m^{-2}\ y^{-1}$
R	$=$		$p[H]$	V

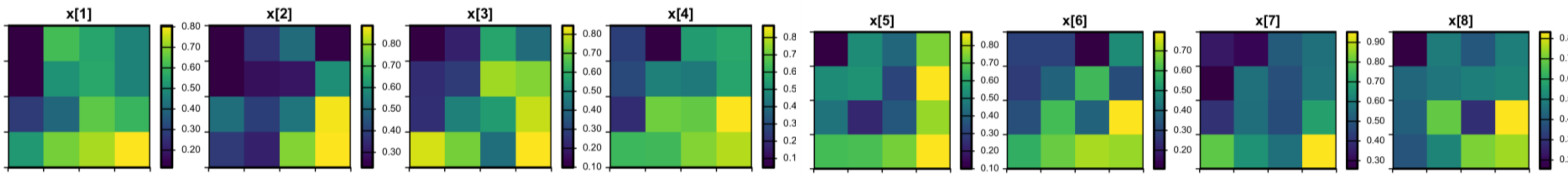
$g\ y^{-1}$		m^2	-	$g\ m^{-2}\ y^{-1}$
R	$=$	Q	$p[H]$	V

Q = area (or # individuals) exposed to the risk factor

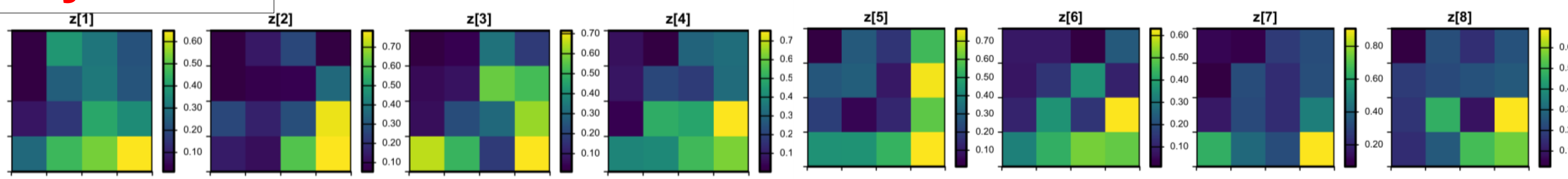
Three-component PRA

x in year 1:8

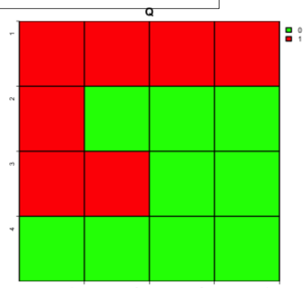
thr = 0.25



z in year 1:8



Q = 7/16



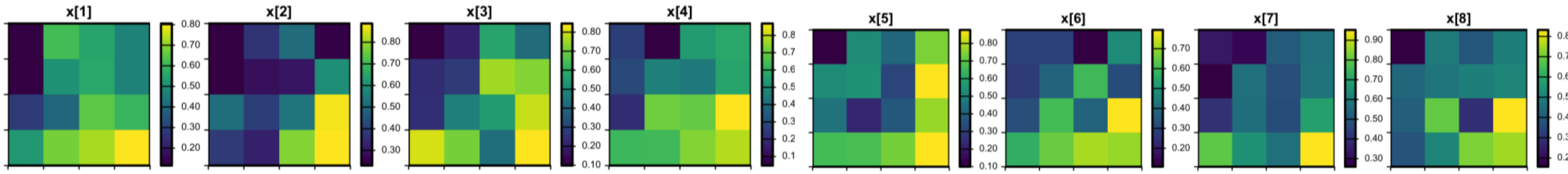
Three-component PRA

```
PRA3 <- function( x=array(dim=c(nlon,nlat,n_t)),
                  z=array(dim=c(nlon,nlat,n_t)), thr.=thr ) {
  ns    <- prod( dim(x)[1:2] ) ; n_t <- dim(x)[3]
  freqH <- function(x,thr.=thr){ sum(x<thr.) }
  n_tH  <- apply(x, c(1,2), freqH)
  siteQ <- which( n_tH > 0, arr.ind=TRUE ) ; nQ <- dim(siteQ)[1]
  Q     <- nQ / ns ; s_Q <- sqrt( Q*(1-Q) / ns )

  xQ    <- sapply( 1:n_t, function(i){x[,,i][siteQ]} )
  zQ    <- sapply( 1:n_t, function(i){z[,,i][siteQ]} )
  PRAQ  <- PRA( c(xQ), c(zQ), thr. )
  pH    <- PRAQ["pH"]   ; V   <- PRAQ["V"]   ; R.Q   <- PRAQ["R"]
  s_pH  <- PRAQ["s_pH"] ; s_V <- PRAQ["s_V"] ; s_R.Q <- PRAQ["s_R"]
  R     <- Q * R.Q
  s_R   <- sqrt( s_Q^2*s_R.Q^2 + s_Q^2*R.Q^2 + Q^2*s_R.Q^2 )

  result            <- c(  Q ,  pH ,  V ,  R ,  s_Q ,  s_pH ,  s_V ,  s_R  )
  names ( result ) <- c( "Q", "pH", "V", "R", "s_Q", "s_pH", "s_V", "s_R" )
  return( result ) }
```

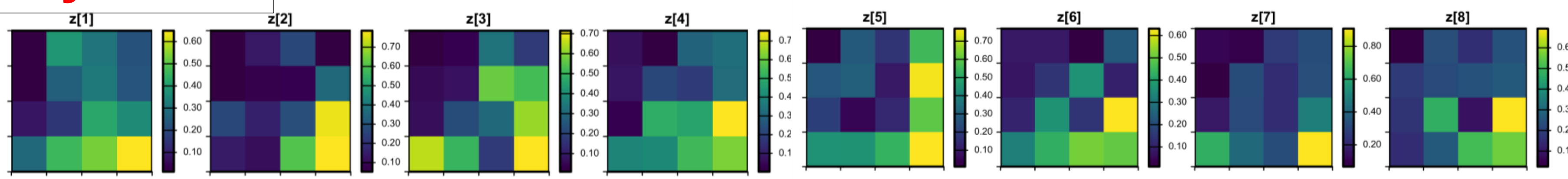
Three-component PRA

x in year 1:8

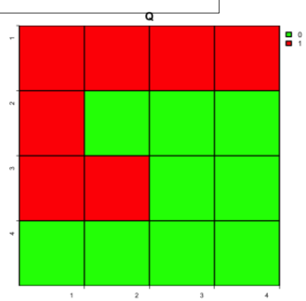
thr = 0.25



z in year 1:8



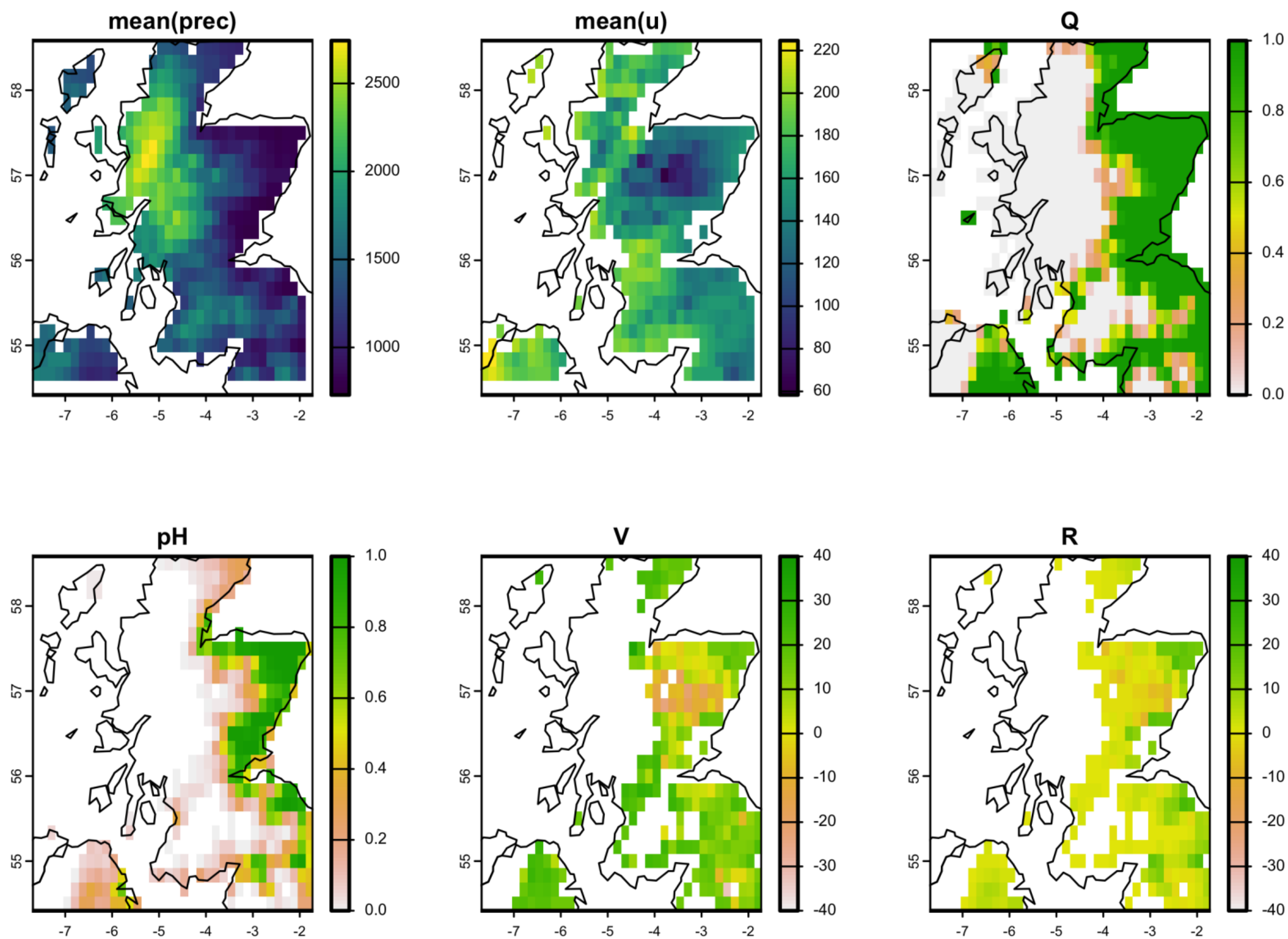
Q = 7/16



Q	pH	V	R
0.438	0.268	0.190	0.022

s_Q	s_pH	s_V	s_R
0.124	0.059	0.020	0.009

Three-component PRA Scotland



Hazardousness can be complicated !

- More than 1 relevant hazard variable
- Effects that depend on hazard time-scale
- Effects that depend on hazard spatial distribution

Hazard modelling:

- Fault-tree analysis (FTA)
- Graphical modelling for p[H] or { p[H_1, .. ,H_n] }
- Copulas
- Extreme-value theory, Generalized extreme value distributions
- Trivariate Gaussian $p[x_1,x_2,z]$ generally too simple …

Liu et al. (2020). A new risk probability calculation method for urban ecological risk assessment

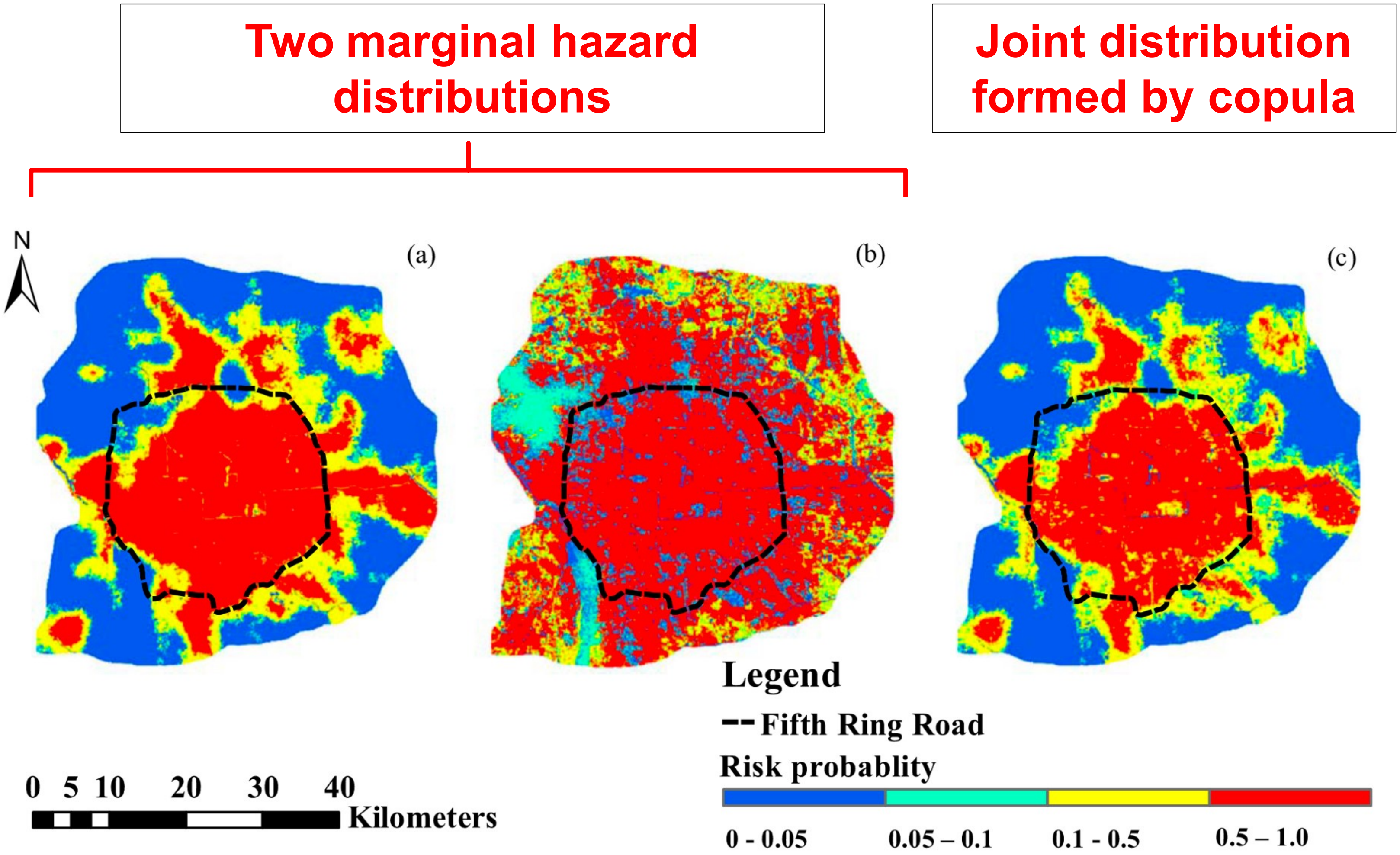
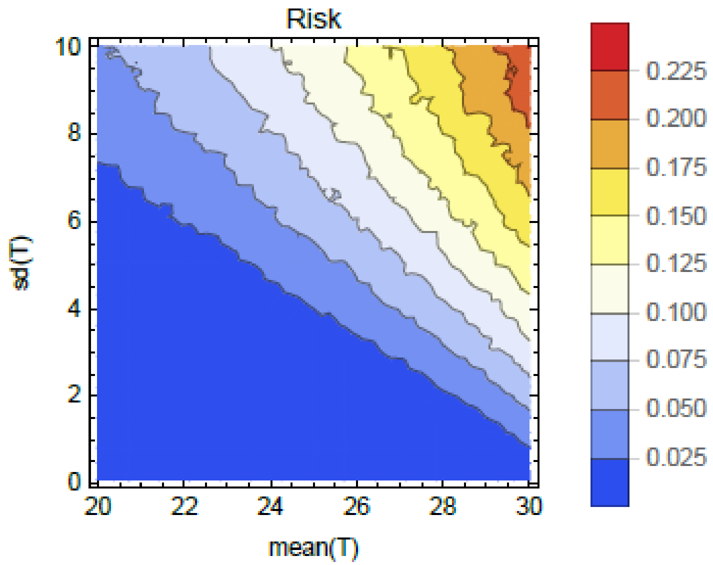
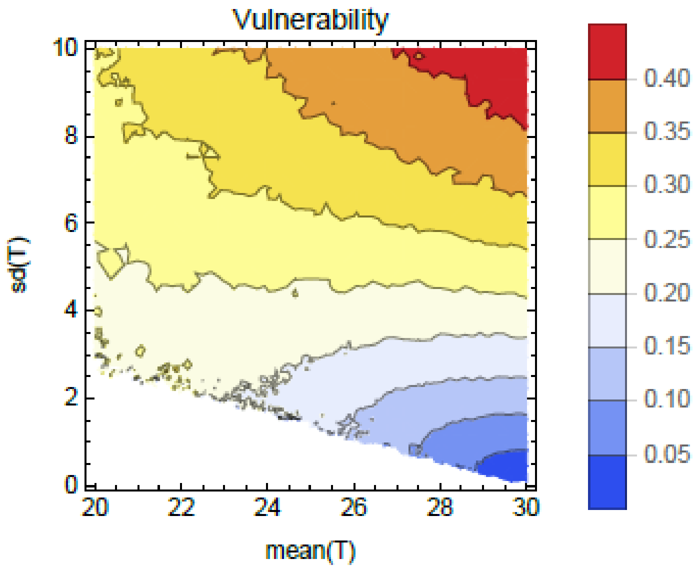
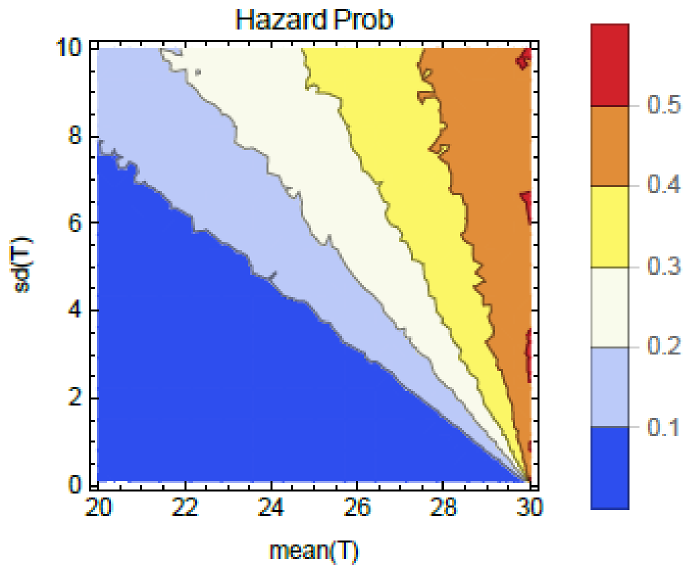


Figure 3. Spatial distribution maps of three risk probability types. (a) Single indicator risk probability that surface runoff exceeds its risk threshold (SRP1). (b) Single indicator risk probability that total nitrogen pollutant load exceeds its risk threshold (SRP2). (c) Multi-indicator risk probability (comprehensive risk probability) that all indicators exceed their respective risk thresholds (MRP).

PRA for heatwaves (thr = 30°C) as f(mean(T),sd(T))







Chen et al. (2020). The collapse points of increasing trend of vegetation rain-use efficiency under droughts

Environ. Res. Lett. **15** (2020) 104072

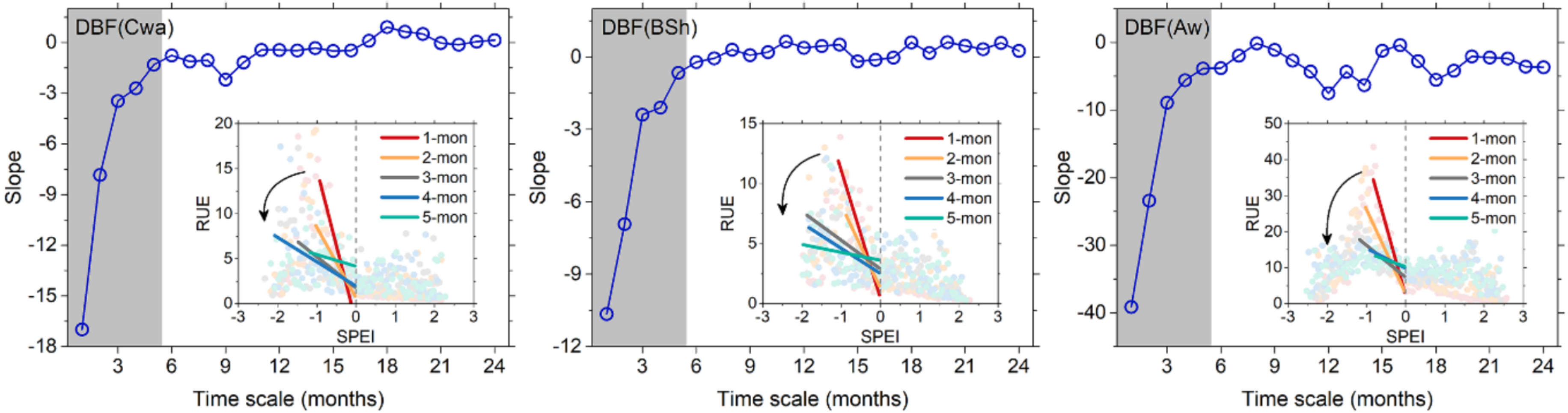
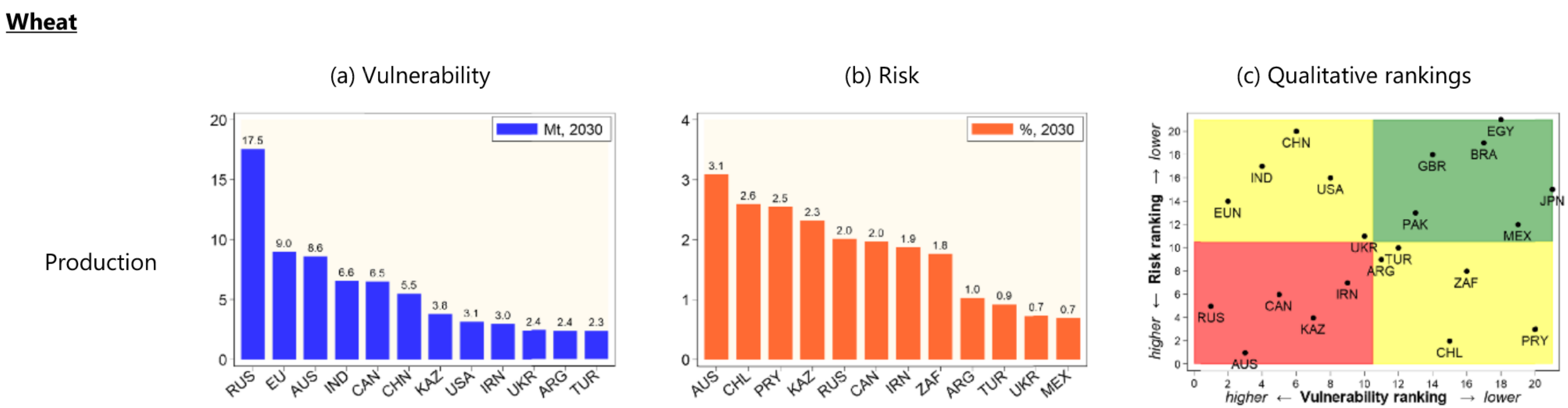


Figure 5. Relationship between SPEI time scale and the slope of linear regression for binned averages of RUE and SPEI during drought (i.e. SPEI < 0) for six typical deciduous broadleaf forest (DBF) biomes. As for the biomes exhibiting apparent tipping point under drought intensity, only the trend of RUE prior to the tipping point is taken into consideration. The inset in each graph shows the relationship between binned averages of RUE and SPEI of multiple time scales (1, 2, 3, 4, and 5 months) corresponding to circles shaded in grey. Colors represent the different SPEI time scales in the six graphs. Arrows indicate the general trend of RUE-SPEI relationship along with the SPEI time scale.

Chatzopoulos et al. (2021). Potential impacts of concurrent and recurrent climate extremes on the global food system by 2030

Environ. Res. Lett. **16** (2021) 124021

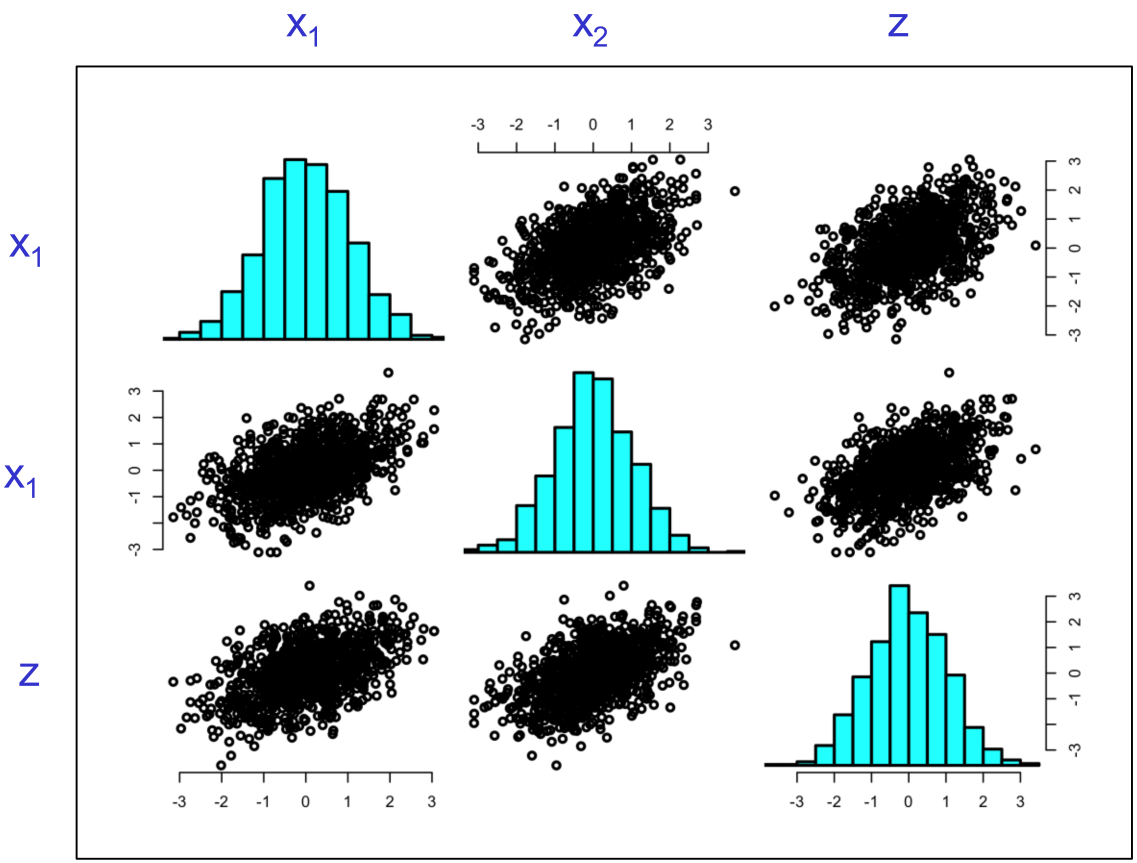


- Concurrent climate extremes → V_{glob} = *f(spatial distrib. of H)*
- Recurrent climate extremes → V_{glob} = *f(temporal distrib. of H)*

Two-hazard sampling-based PRA

```
PRAi <- function( xz, thr=c(0,0) ) {
  x1      <- xz[,1]  ; x2 <- xz[,2] ; z <- xz[,3]
  n_c     <- 2^2 - 1 ; n  <- length(x1)
  H       <- vector("list",n_c)
  n_H     <- pH <- V <- R <- s_pH <- s_V <- s_R <- rep(NA,n_c)

  H[[1]]  <- which(x1 <  thr[1] & x2 <  thr[2]) ; n_H[1] <- length(H[[1]])
  H[[2]]  <- which(x1 <  thr[1] & x2 >= thr[2]) ; n_H[2] <- length(H[[2]])
  H[[3]]  <- which(x1 >= thr[1] & x2 <  thr[2]) ; n_H[3] <- length(H[[3]])
  NotH    <- which(x1 >= thr[1] & x2 >= thr[2]) ; n_NotH <- length(NotH)

  pH      <- n_H / n           ; s_pH      <- sqrt( pH*(1-pH) / n )
  Ez_NotH <- mean( z[NotH] ) ; s_Ez_NotH <- sqrt( var(z[NotH] ) / n_NotH )
  for(i in 1:n_c) {
    Ez_Hi   <-       mean( z[ H[[i]] ] )
    s_Ez_Hi <- sqrt( var ( z[ H[[i]] ] ) / n_H[i] )
    V[i]    <- Ez_NotH - Ez_Hi
    s_V[i]  <- sqrt( s_Ez_NotH^2 + s_Ez_Hi^2 ) }
  R       <- pH * V
  s_R     <- sqrt( s_pH^2 * s_V^2 + s_pH^2 * V^2 + pH^2 * s_V^2  )

  R.sum   <- sum(R) ; pH.sum <- sum(pH) ; V.wsum <- R.sum / pH.sum
  return( list( sum = c( pH.sum=pH.sum, V.wsum=V.wsum, R.sum=R.sum ),
                cat = cbind( 1:3, pH, V, R, s_pH, s_V, s_R ) ) ) }
```

Two-hazard PRA (Trivariate Gaussian example)



Mean vector

0.006	0.001	-0.024

Covariance matrix

1.000	0.499	0.493
0.499	1.000	0.519
0.493	0.519	1.000

```
>   PRAi(xz_G3,c(0,0))$cat
        pH     V     R  s_pH   s_V   s_R
[1,] 1 0.342 1.226 0.419 0.015 0.071 0.030
[2,] 2 0.160 0.625 0.100 0.012 0.083 0.015
[3,] 3 0.159 0.687 0.109 0.012 0.088 0.016
```

```
> PRAi(xz_G3,c(0,0))$sum
pH.sum V.wsum  R.sum
 0.661  0.951  0.629
```

EXERCISE

- Create the following dataset:

```
set.seed(1)
  n  <- 1e2
  x1 <- rbeta( n, 3, 3 ) ; x2 <- rbeta( n, 3, 3 )
  z  <- as.integer( x1 >= 0.5 | x2 >= 0.5 )
  xz <- cbind( x1, x2, z )
```

- Study the dataset. R-command: `pairs(xz)`
- Run `PRAi` on this dataset using `thr=c(0.5,0.5)`
- Explain the results, especially for V and s_V

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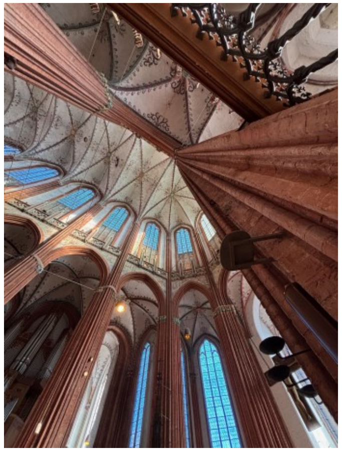
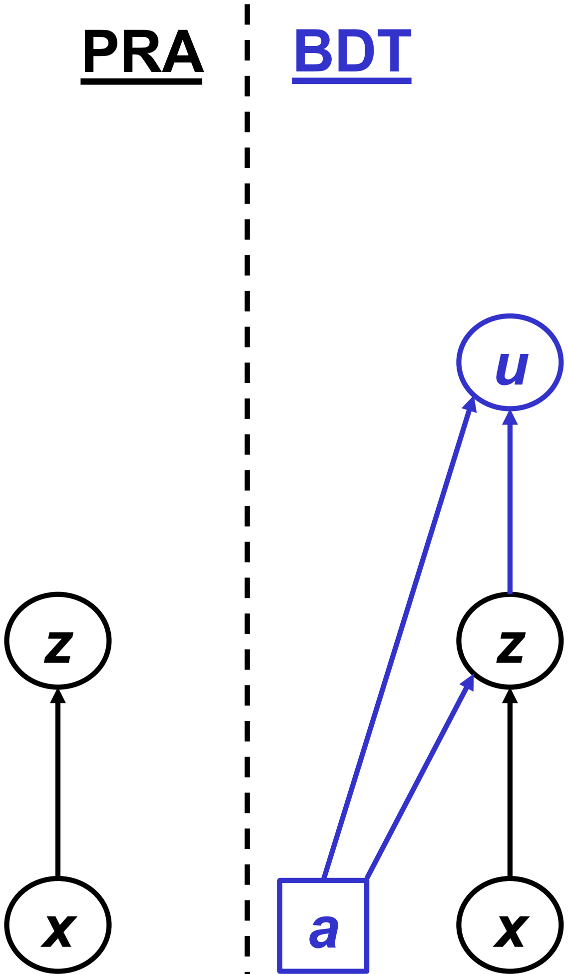
4.
Introduction to BDT



Bayesian Decision Theory (BDT)

- BDT is the application of probability theory to decision making in situations of uncertainty
- Two concepts: probability & utility
- Three ingredients:
  1. List or continuum of possible actions $a \in A$
  2. List or continuum of external conditions $x \in X$
     - These are uncertain: we have p[x]
  3. A utility function u(a,x)
     - Our preference: high values of u(a,x)
     - Our decision: action a* that maximises $\bar{u}(a) = E[u(a,x)]$
- BDT is related to but different from PRA
- Our examples are of BDT in (forest) management decision-making
- We will not discuss other well-known applications of BDT:
  - Parameter estimation: cost = minus utility = f(estimation error) (Bernardo & Smith 2000)
  - Causal analysis: causality = impact of intervention on utility (Dawid 2021)

Graphical models: From PRA to BDT



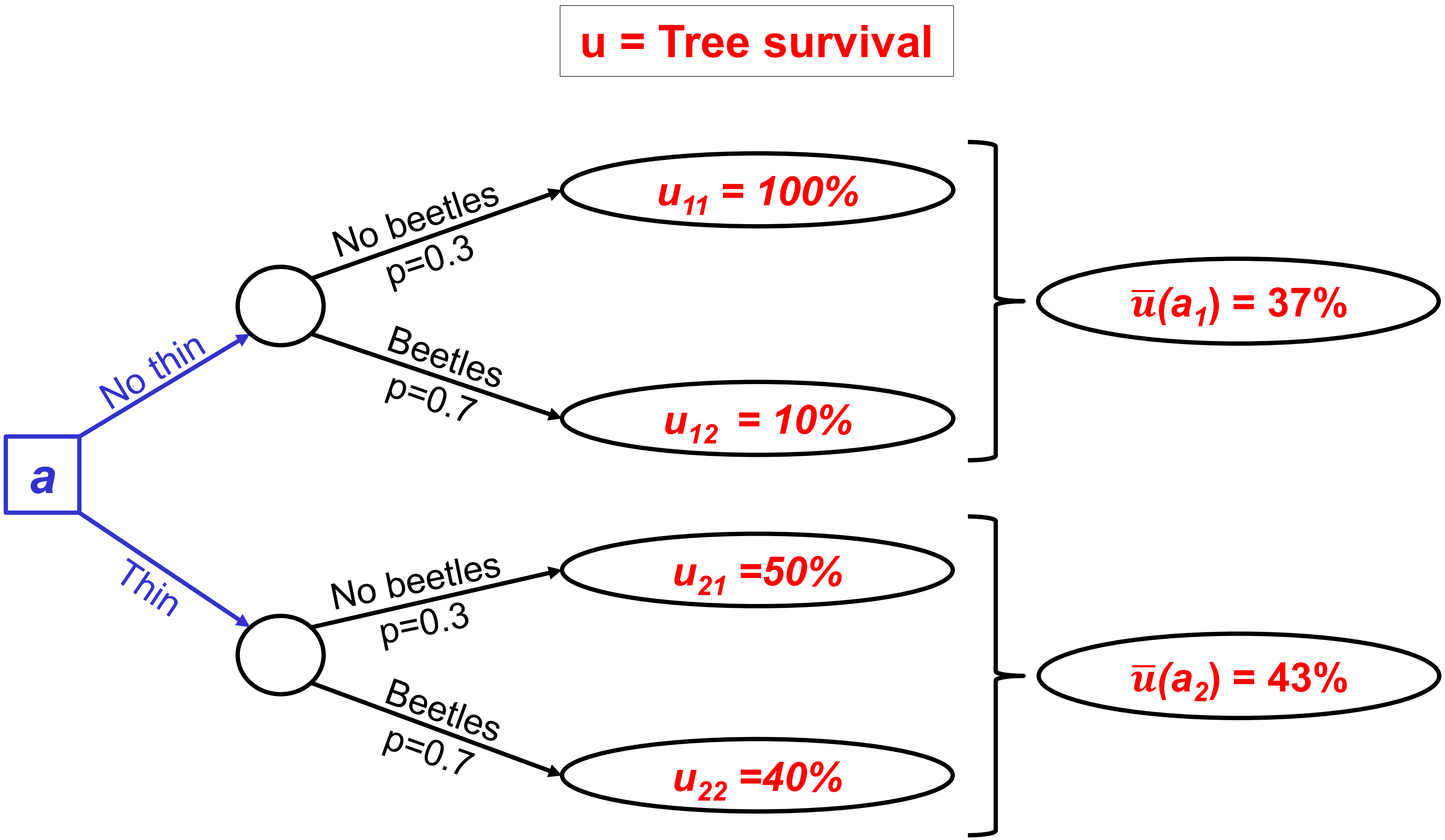
$$\overline{u}(a) = E[u(a)] = \sum p[x]\, u(a,x)$$

$$\overline{u}(a^*) = \max_{a \in A} \overline{u}(a)$$

p[x,z] = p[x] p[z|*x*]

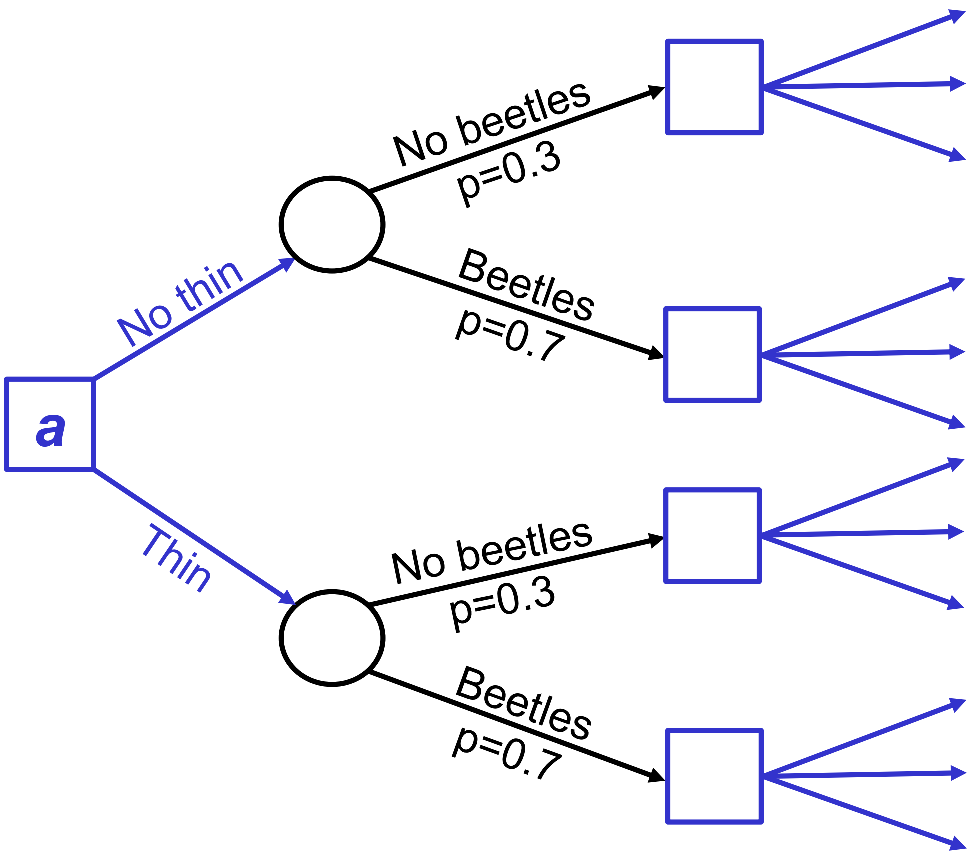
p[x,z,u|a] = p[x] p[z|a,*x*] p[u|a,z]

Decision tree

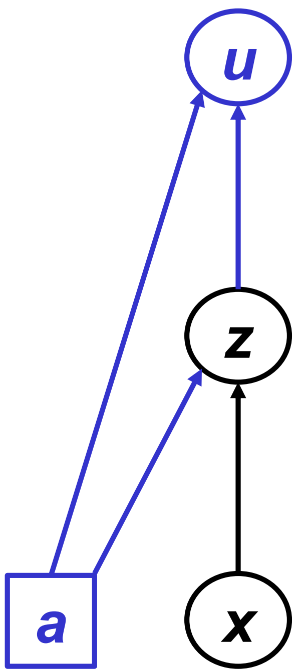


We choose a_2 !

Sequential decisions



Utility matrix for discrete u(a,x)



	$x = x_1$	$x = x_2$	$\bar{u}(a) = \sum p[x]\, u(a,x)$
$a = a_1$	$u(a_1,x_1)$	$u(a_1,x_2)$	$p_1\, u(a_1,x_1) + p_2\, u(a_1,x_2)$
$a = a_2$	$u(a_2,x_1)$	$u(a_2,x_2)$	$p_1\, u(a_2,x_1) + p_2\, u(a_2,x_2)$

% Tree survival	No beetles	Beetles	$\bar{u}(a)$ if $p[x = x_2 = \text{Beetles}] = 0.7$
No thin	100%	10%	0.3×100 + 0.7×10 = 37%
50% thin	50%	40%	0.3×50 + 0.7×40 = 43%

We choose a_2 !

Value of Information: Impact on expected utility

Only prior information p[x]:

$$\overline{u}(a^*) = \max_{a \in A} \sum p[x]\, u(a,x)$$

Specific information y:

$$\overline{u}(a^*_y) = \max_{a \in A} \sum p[x|y]\, u(a,x)$$

Information yet to be received Y = {y}:

$$\overline{u}(a^*_Y) = \sum_{y \in Y} p[y]\, \overline{u}(a^*_y)$$

$$= \sum_{y \in Y} \max_{a \in A} \sum p[x]\, p[y|x]\, u(a,x)$$

'Perfect' information *so you can always (for every x) choose the best action:*

$$\overline{u}(a^*_I) = \sum p[x] \max_{a \in A} u(a,x)$$

Value of Information in the tree beetle example

$$\overline{u}(a^*) = 43\%$$

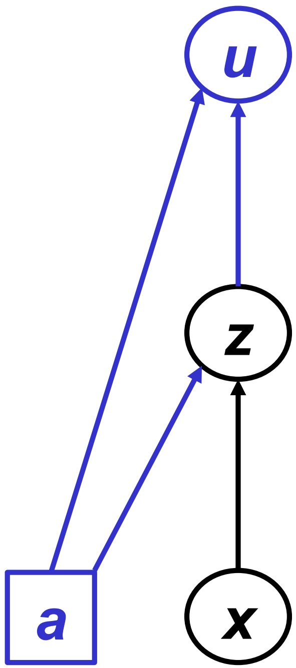
$$\overline{u}(a^*_Y) = \sum_{y \in Y} \max_{a \in A} \sum p[x]\, p[y|x]\, u(a,x)$$

If y = {y_1,y_2} *and*
$p[y_1|x_1] = p[y_2|x_2] = 0.6$

$$= \max(20.8, 20.2) + \max(16.2, 22.8) = 43.6\%$$

$$\overline{u}(a^*_I) = \sum p[x] \max_{a \in A} u(a,x)$$

$$= 0.3 \times 100 + 0.7 \times 40 = 58\%$$



$VoI_{partial}$ = 43.6 – 43 = 0.6%

$VoI_{perfect}$ = 58 – 43 = 15%

EXERCISE - Value of bad Information …

What would happen to $VoI_{partial}$ if $p[y_1|x_1] = p[y_2|x_2] = 0.5$?

$$\overline{u}(a_Y^*) = \sum_{y \in Y} \max_{a \in A} \sum p[x]\, p[y|x]\, u(a,x)$$

$$= 2 \times \max_{a \in A} \sum p[x]\, \frac{1}{2}\, u(a,x)$$

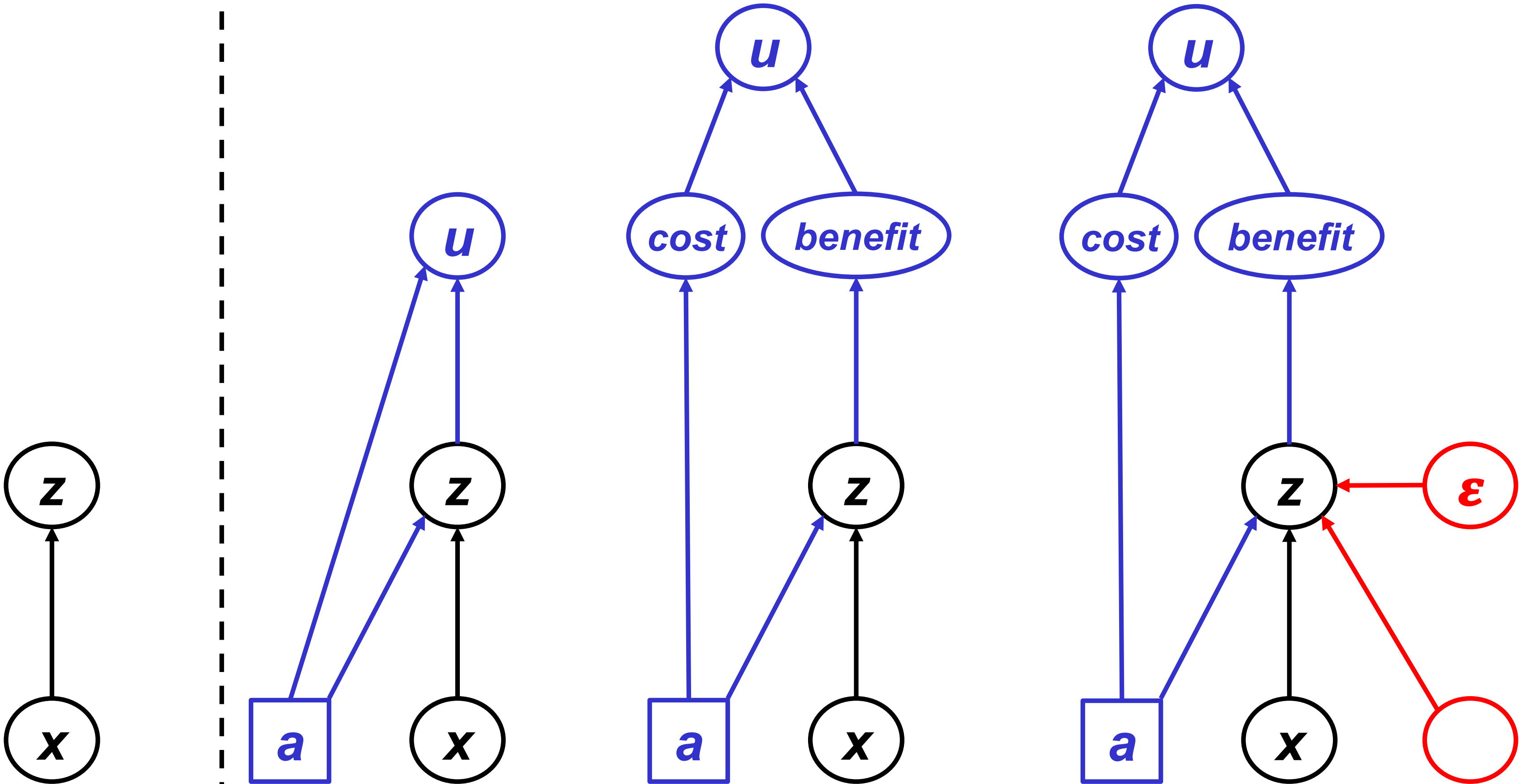
$$= \max_{a \in A} \sum p[x]\, u(a,x)$$

$$= \overline{u}(a^*)$$

$$\Longrightarrow \quad VoI_{partial} = \overline{u}(a_Y^*) - \overline{u}(a^*) = 0$$

Graphical models: From PRA to BDT

PRA | **BDT**

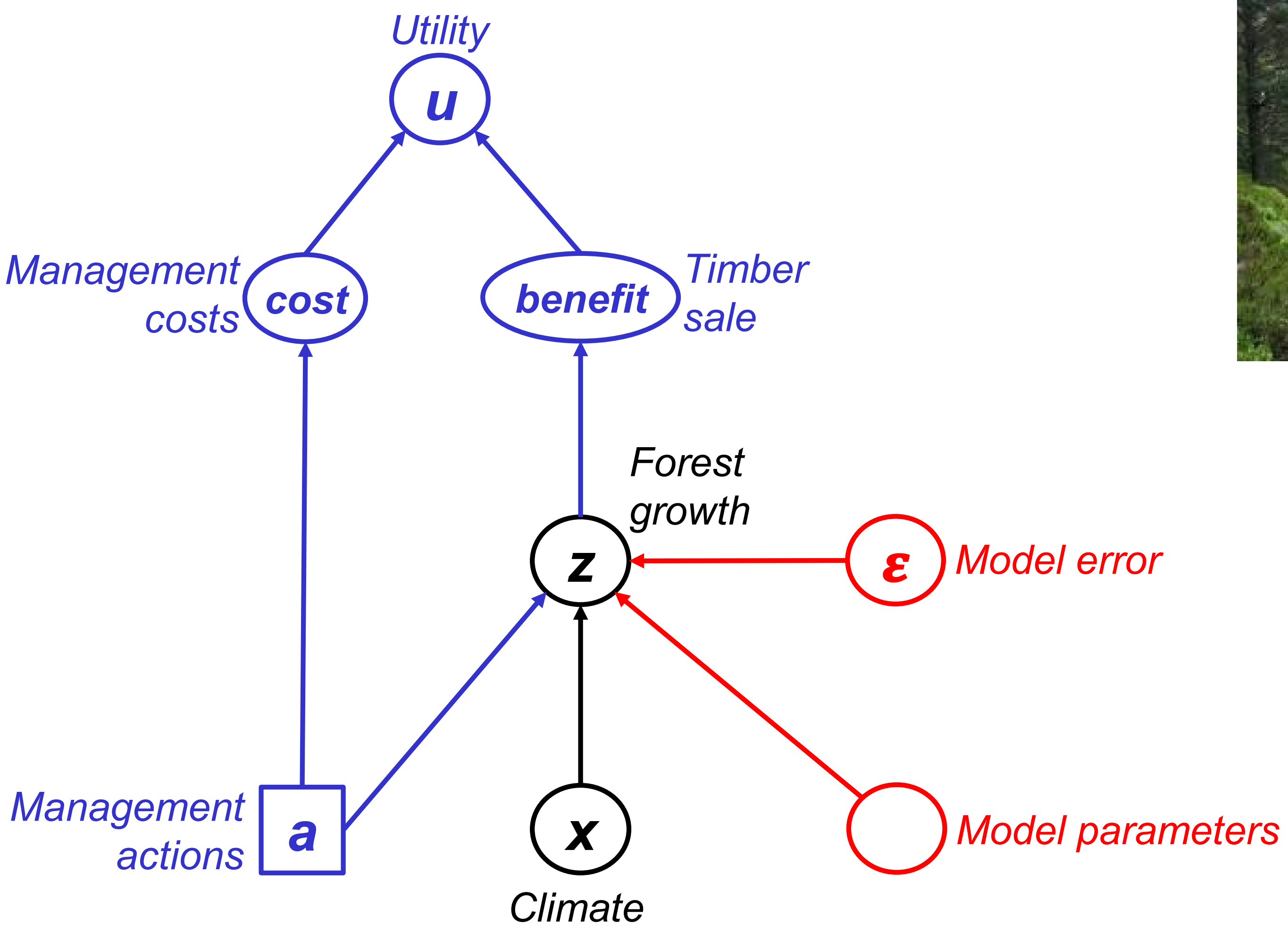


$\mathbf{p[x,z]} =$
$p[x]\,p[z|x]$

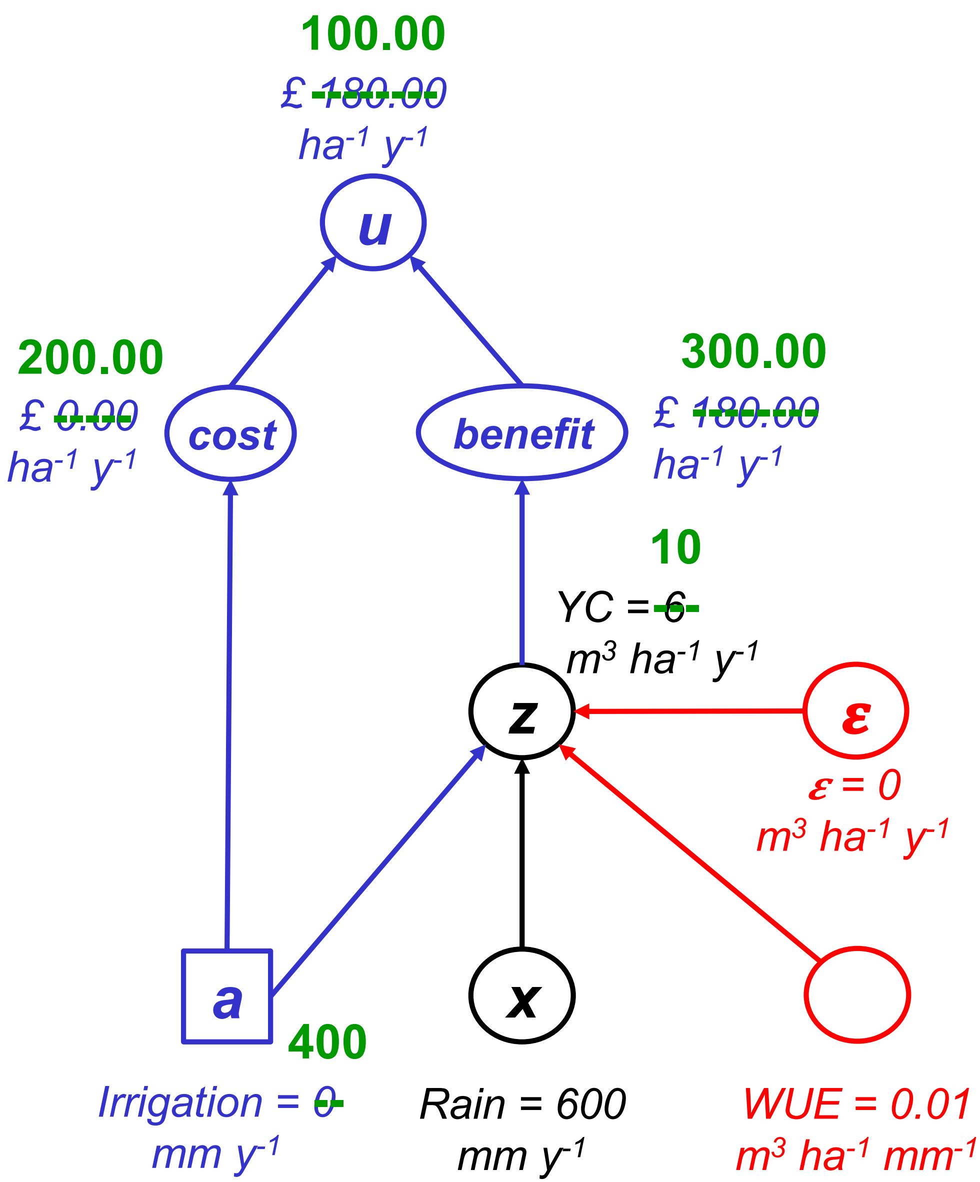
$\mathbf{p[x,z,u|a]} =$
$p[x]\,p[z|a,x]\,p[u|a,z]$

$\mathbf{z} = f(a,x,\vartheta) + \varepsilon$
$p[x],\ p[\vartheta],\ p[\varepsilon]$

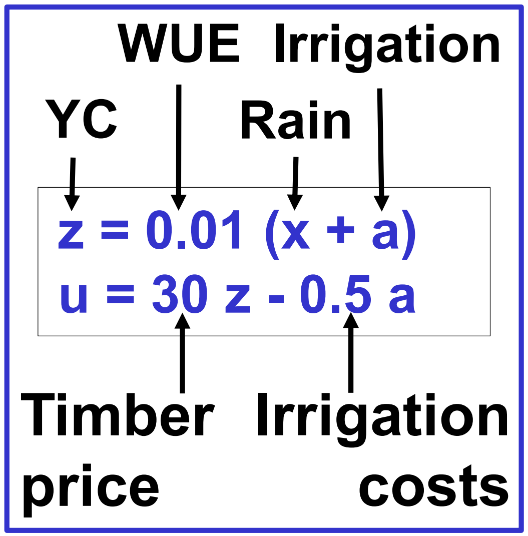
BDT for forestry



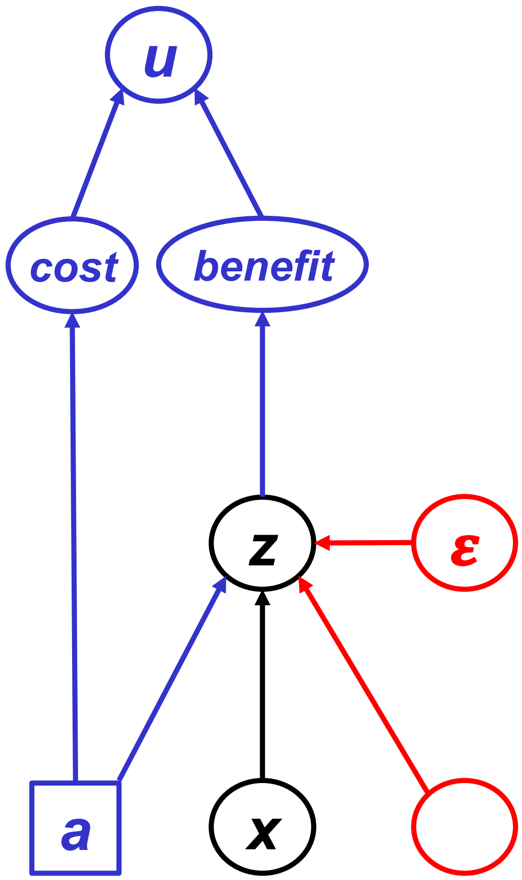
Two realisations of the network



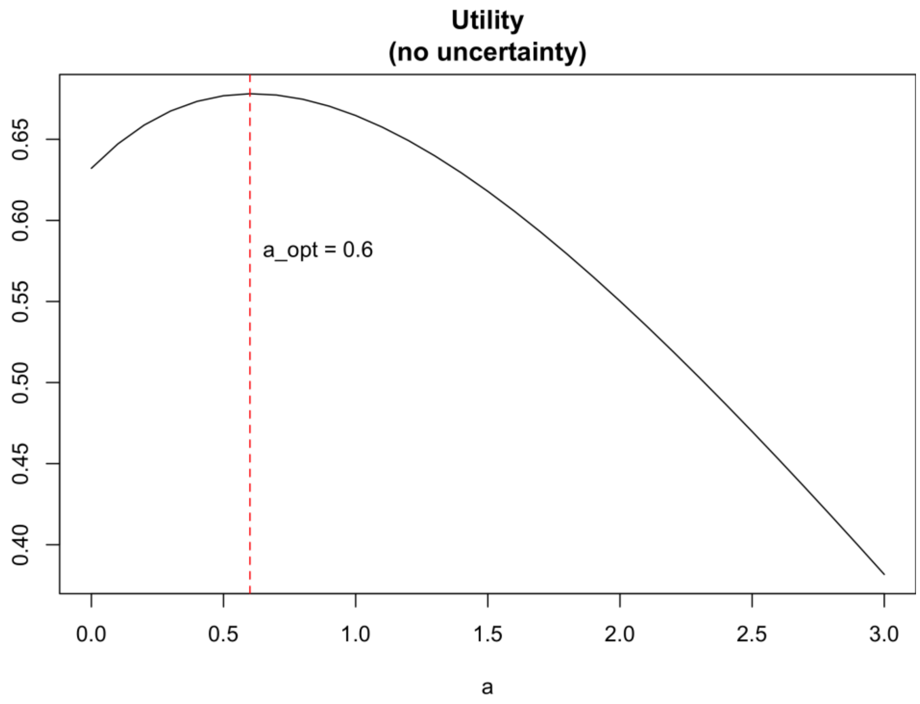
Model



BDT with continuous nonlinear z-response function

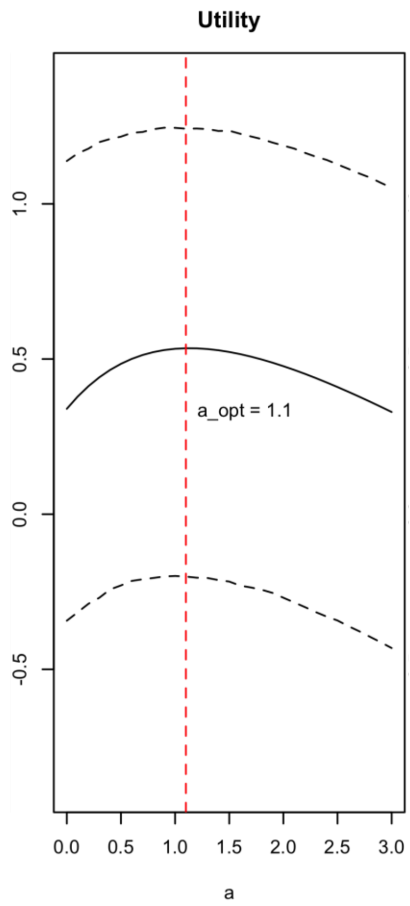



```
u <- function( a, x=1, t=1, e=0, ka=0.2, kz=1 ) {
  z    <- t*(1-exp(-a-x)) + e
  cost <- ka*a ; benefit <- kz*z
  return( benefit - cost ) }
```



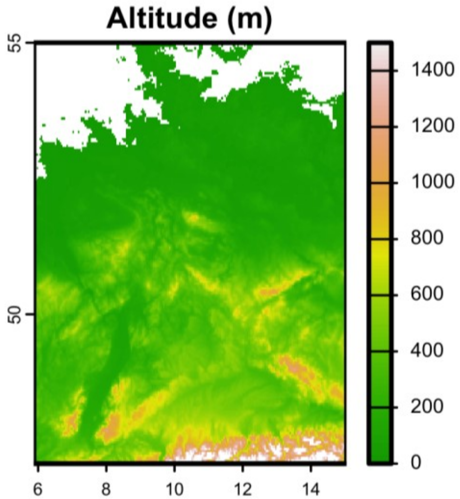
BDT with continuous nonlinear z-response function

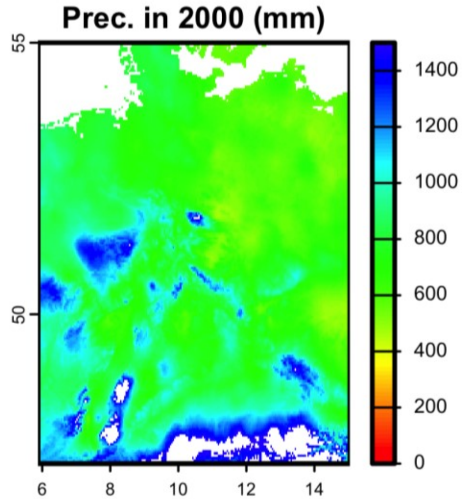
$$
\begin{aligned}
x &\sim N[1,1], \\
t &\sim N[1,0.5], \\
e &\sim N[0,1], \\
ka &\sim U[0.1,0.3], \\
kz &\sim U[0.5,1.5].
\end{aligned}
$$



Medium uncertainty

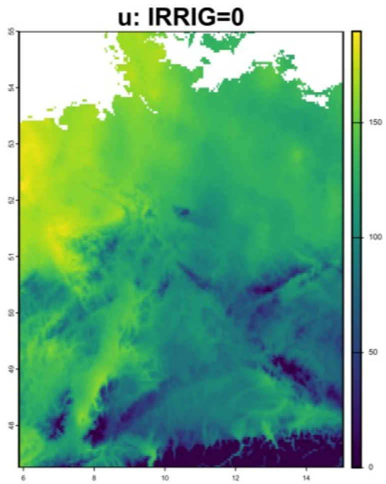
A spatial BDT example

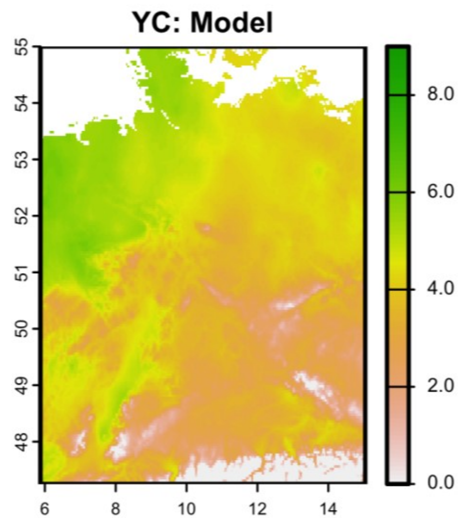


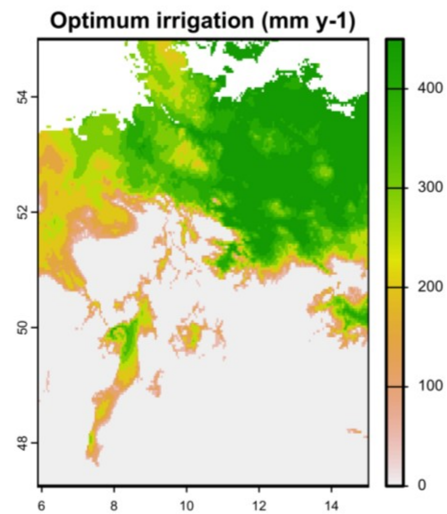


Models for z = forest yield class (m^3 ha^{-1} y^{-1}), and u:

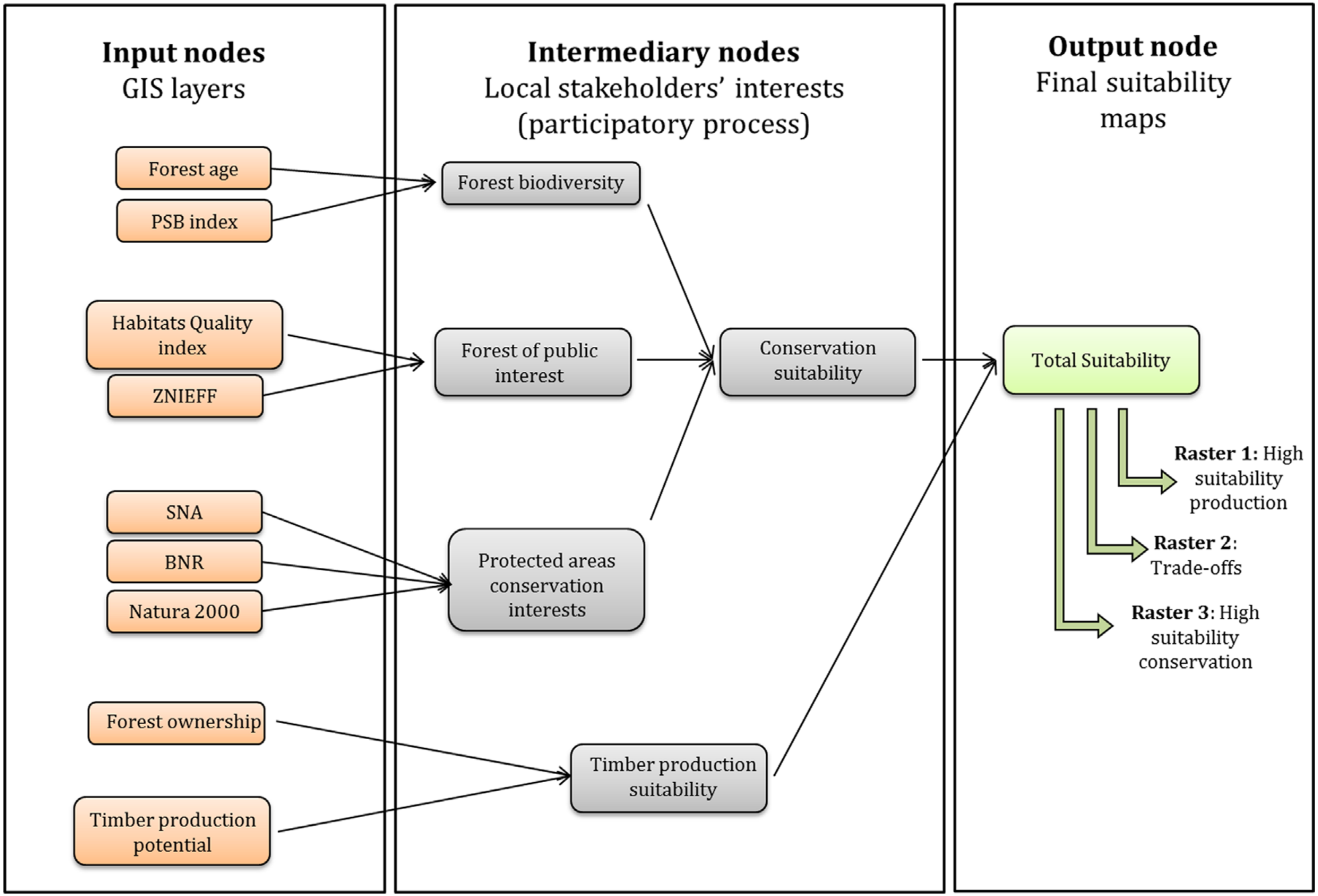
```
YC = 10 * (1-exp(-water/1000)) * (1-altitude/1000)
u  = 30 * YC - 0.1 * IRRIG
```







Gonzales-Redin et al. (2016)



Gonzalez Redin, J. et al. (2016). Spatial Bayesian belief networks as a planning decision tool for mapping ecosystem services trade-offs on forested landscapes. Environmental Research 144: 15-26.

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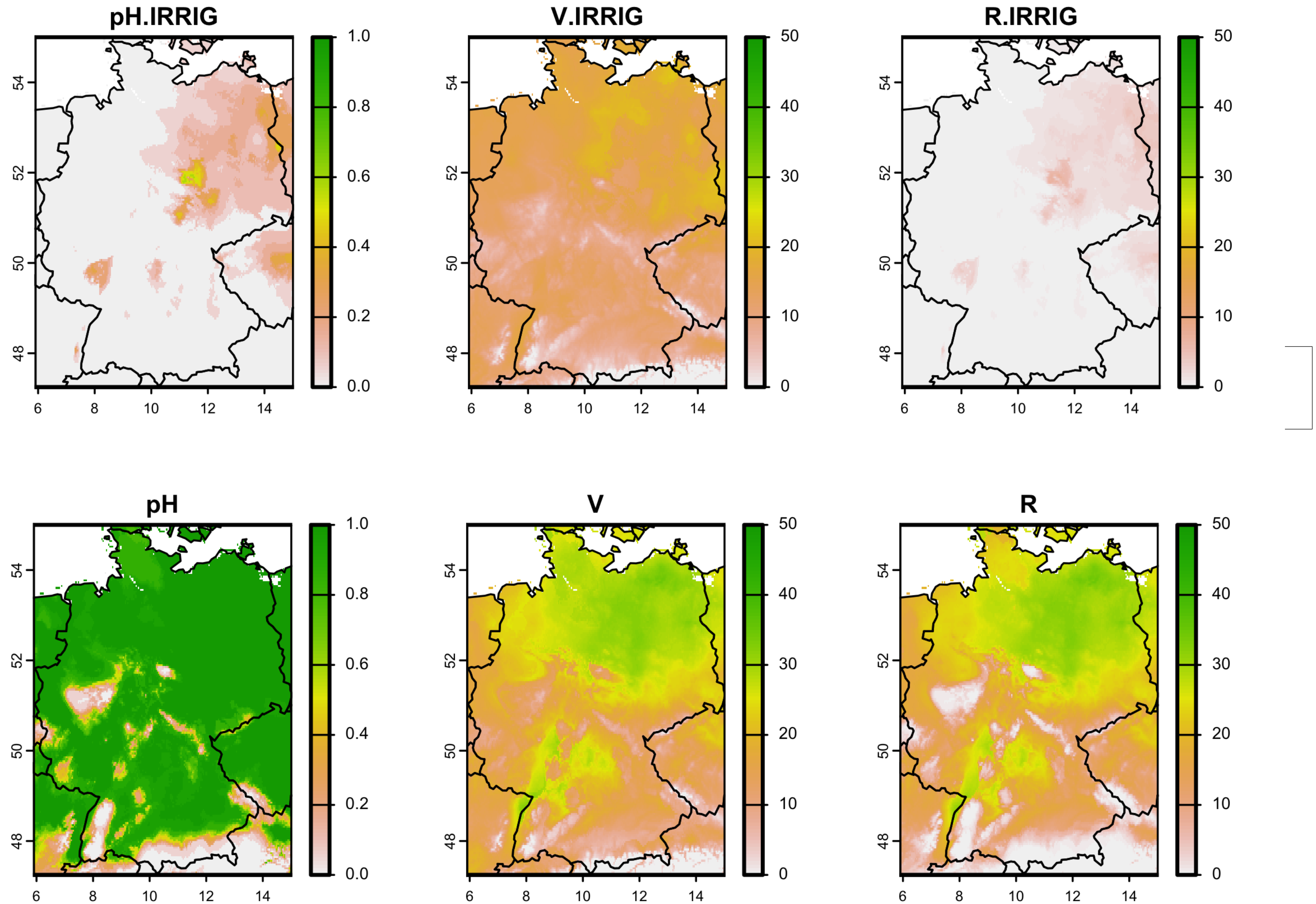
5.
Links between PRA and BDT

A spatial BDT example



Maximizing utility vs. minimizing risk

PRA: irrigation reduces R to nearly zero everywhere…

Why did the BDT then only suggest irrigation in the North?

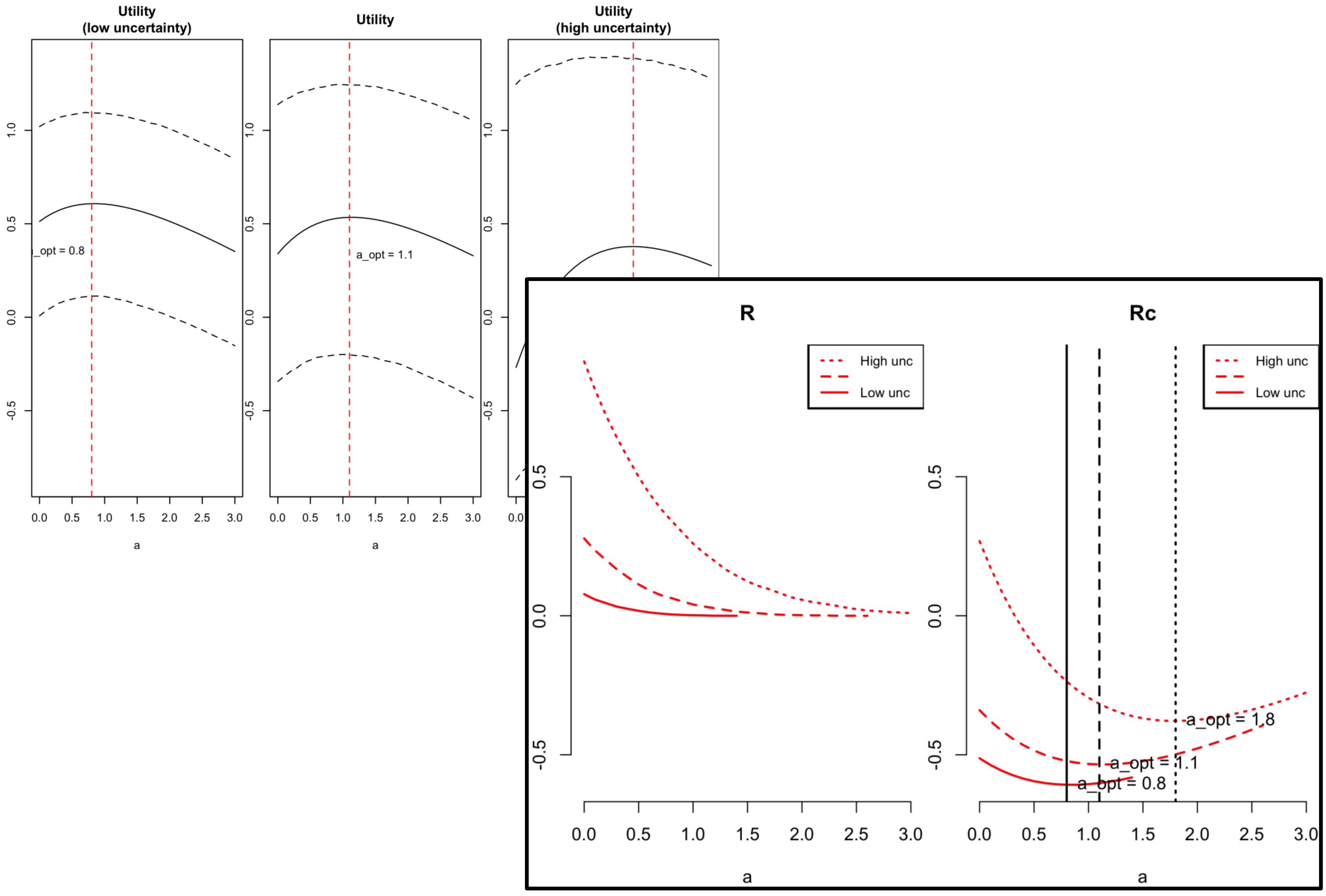
$$R = E[u|\neg H] - E[u]$$

⇒ minimizing R is not the same as maximizing E[u].

We define *'Risk corrected for costs and benefits'* as:

$$R_c = R - E[u|\neg H]$$
$$= R + E[ka|\neg H] \times a - E[ky \times y|\neg H]$$

BDT with continuous nonlinear z-response function



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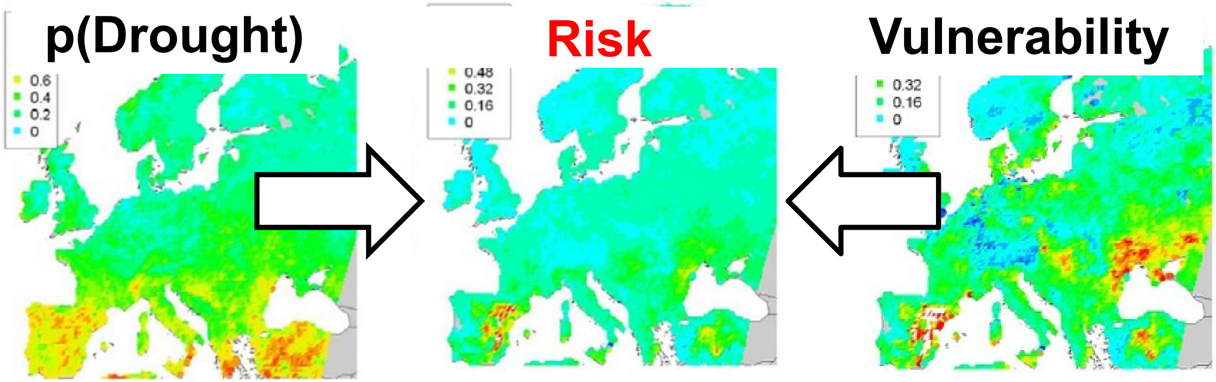
6.
General discussion

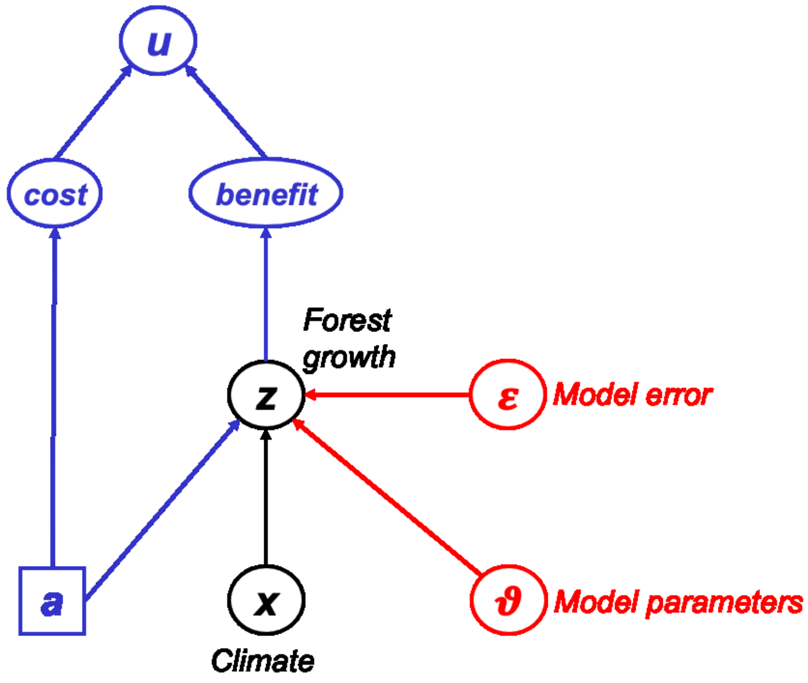
Theory development: PRA & BDT

Probabilistic Risk Analysis (PRA)

Risk = Expected loss = probability(Hazard) * Vulnerability



Bayesian Decision Theory (BDT)

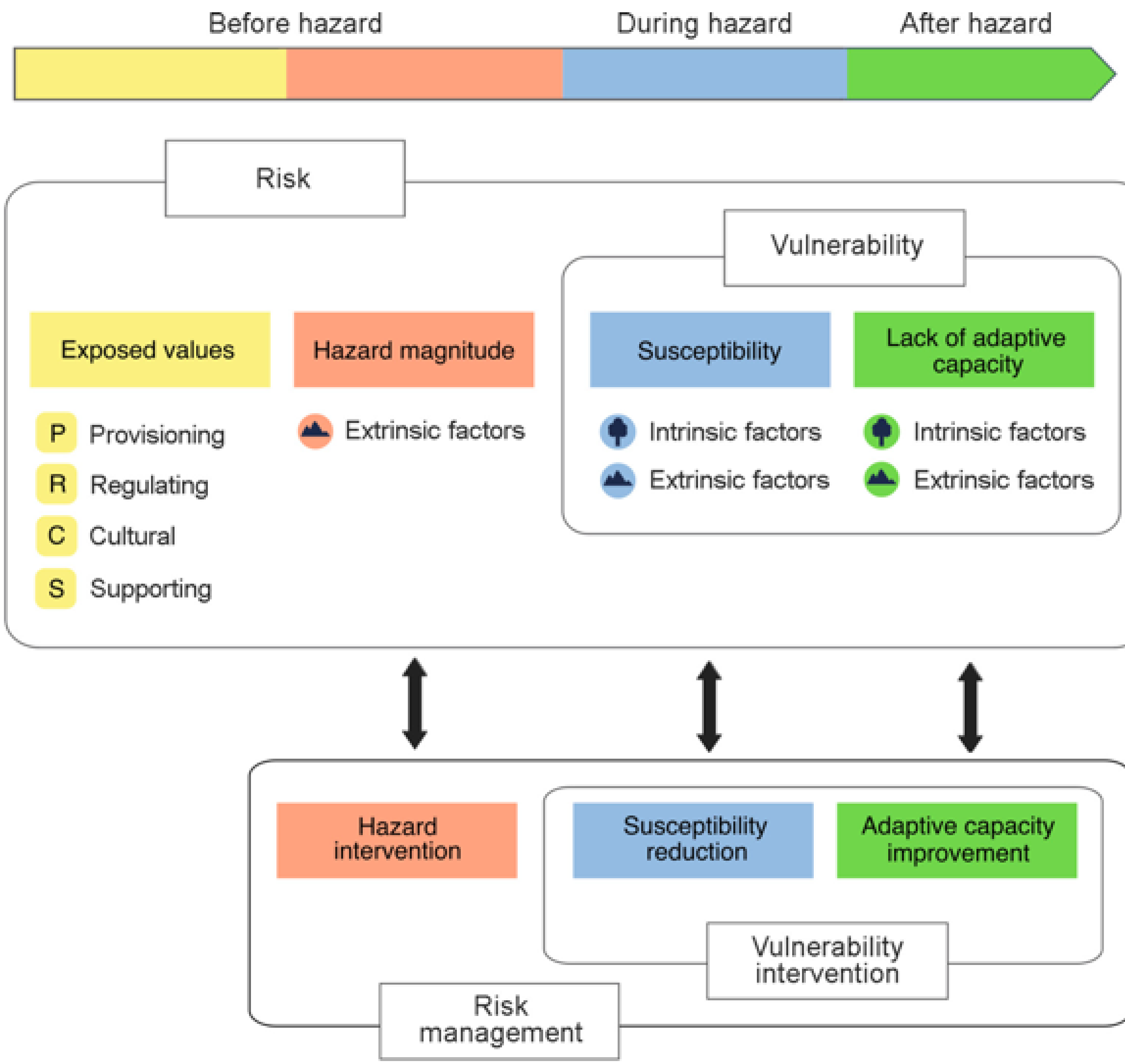


Two questions …

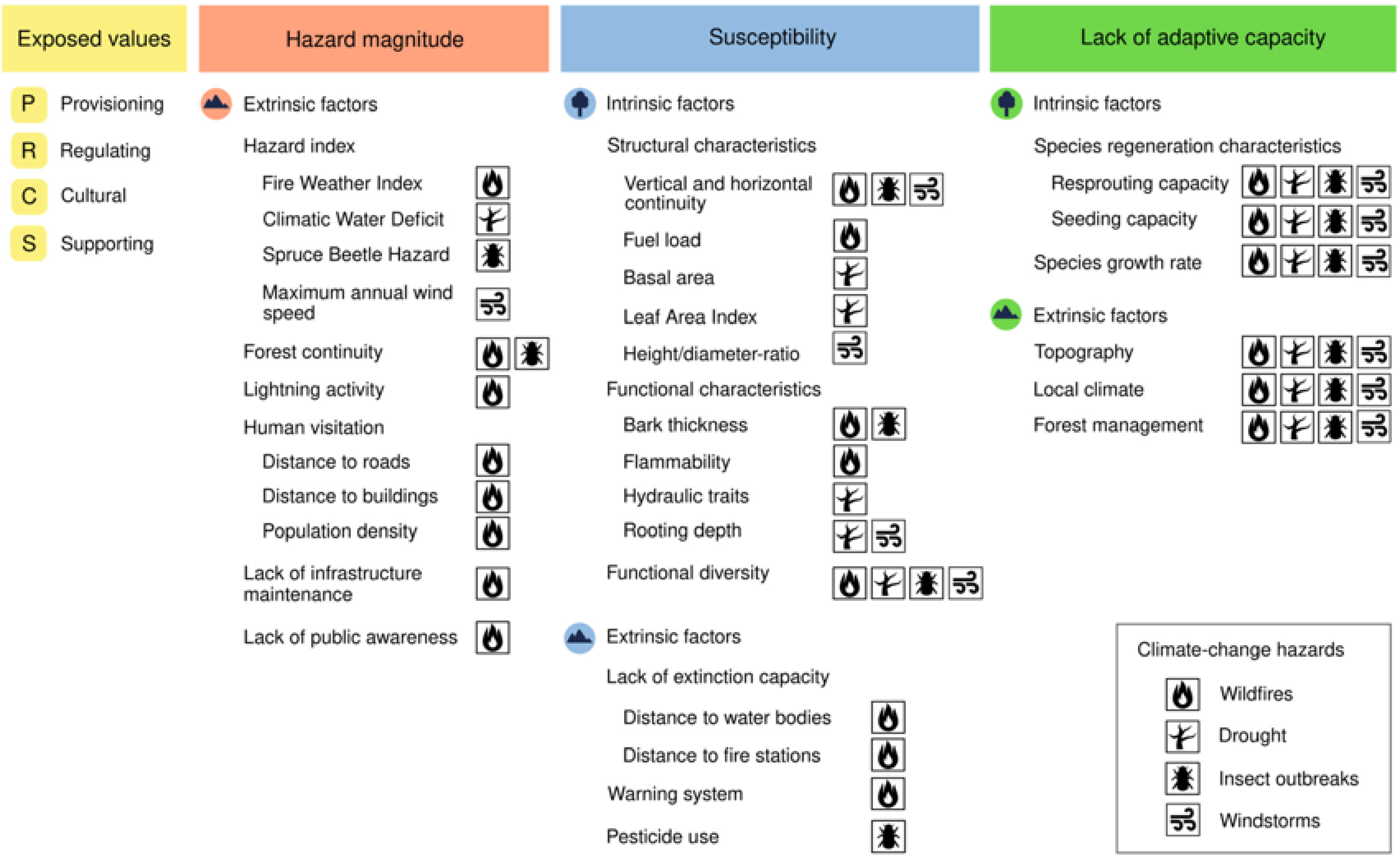
Main questions to all:

- ***What is missing from these lectures and/or VO & B (2022)?***
- ***Which of the methods could you use in your own work?***

Complications: Analysis of z, H and V (Lecina-Diaz et al. 2020)



Complications: Analysis of z, H and V (Lecina-Diaz et al. 2020)



Data & Process-Based Modelling (PBM)

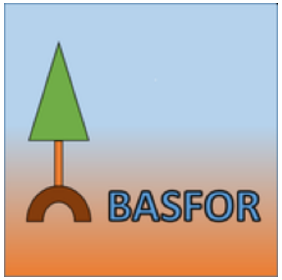
Data

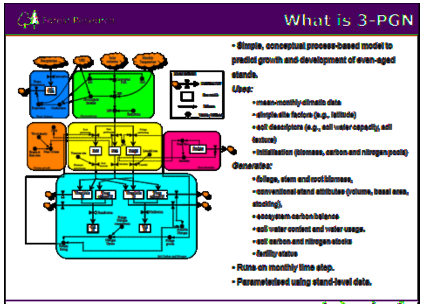
- Performance of conifer spp. in UK, Spain, Finland
- Environmental conditions (soil, past & future climate)
- Forest management

- Model drivers
- Model test data
- Model calibration data

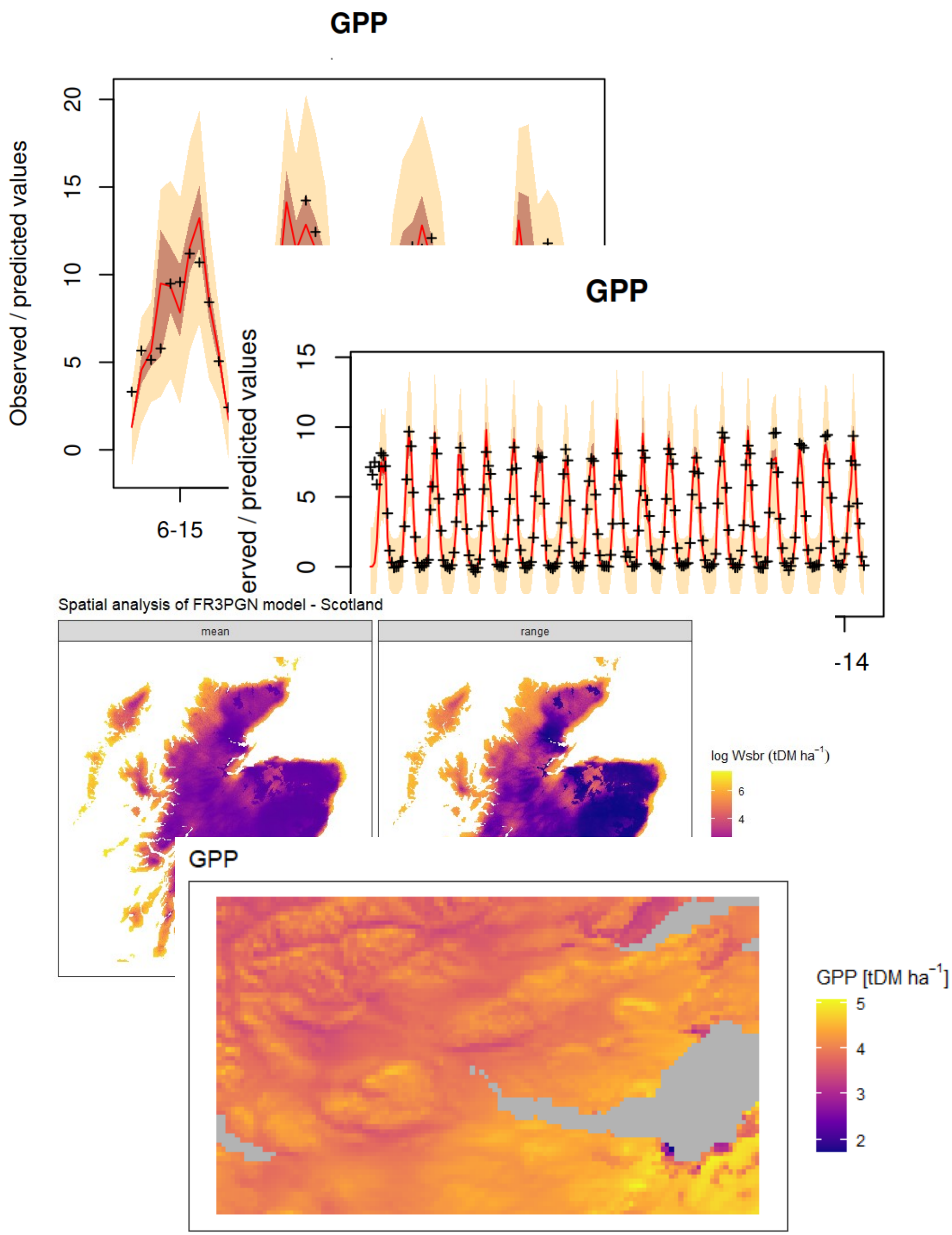
PBM

- BASFOR, 3PGN, …
- Quantifying past & future forest performance
- Analysing forest vulnerability to drought
- Analysing impacts of management



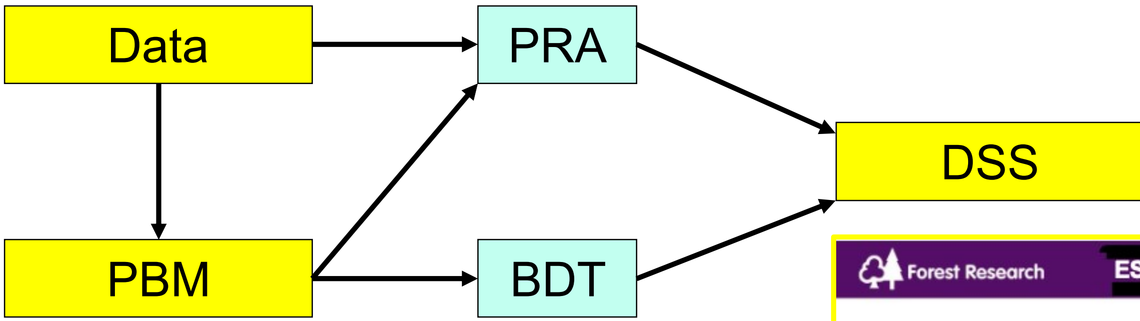


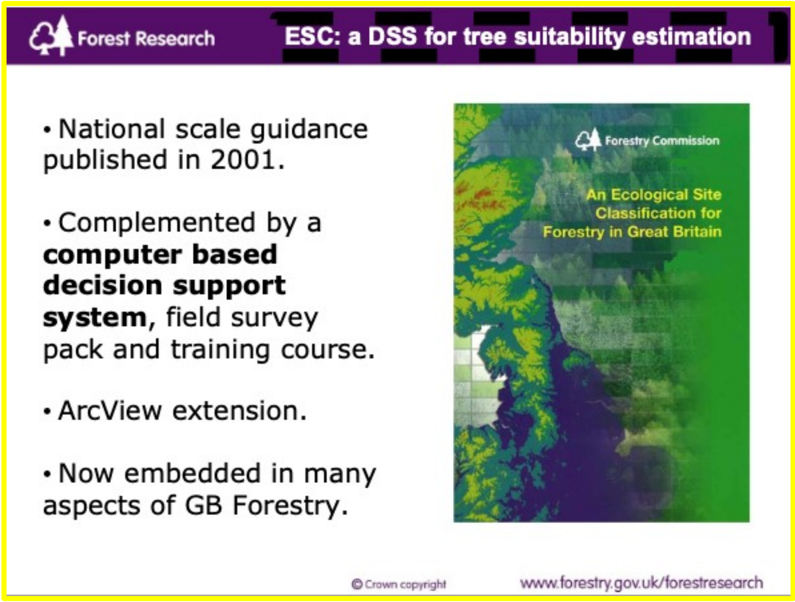
Process-models for forest drought vulnerability



- Climate models (GCMs)
- Process-based forest models (BASFOR, 3PGN, …)
- Identify key factors that drive present and future forest drought vulnerability
- Forest data to test and calibrate the models
- Bayesian Calibration to reduce uncertainties in model parameters
- Bayesian Model Comparison to reduce uncertainty in model error

Project flowchart & Decision Support System (DSS)





Need to know where/what to plant to ensure 'climate-smart' forestry

Issues to discuss

1. Which type of PRA to choose?
   - Nature of x and z, Main questions of interest, Available data and model output
2. Data needs and computational demand of PRA
   - Data needs and computational demand: model-based > distr.-based > sampling-based
   - Data needs: increasing when p[x,z] varies over time and/or space
   - Need for 'extreme' data
   - Model error = f(x)
3. BDT
   - Agreed-upon utility function?
4. Computational demand of BDT
   - High because: 1. Quadruple iteration (actions, parameters, space, time), 2. Slow models
5. PRA as a tool for simplifying and elucidating BDT
   - PRA can be decomposed ⇒ easier to explain? BDT more relevant to decision-making
6. Parameter and model uncertainties
   - Bayesian calibration and Bayesian model comparision: effective but data-hungry and slow
7. Modelling, PRA and BDT for forests
   - Complex system requiring multi-hazard, multi-benefit approach
8. Spatial statistics
   - Spatially correlated hazards ⇒ Regional risk is not the sum of local risks. [Hochrainer-Stigler et al. (2019) used a PBM, EVT & copulas to upscale drought risk in space & time.]

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Appendices

Condition for V being threshold-independent

$$\frac{dV}{d\,thr} = \frac{d\left(E[z|x \geq thr] - E[z|x < thr]\right)}{d\,thr}$$

$$= p[thr]\left\{\frac{E[z|x \geq thr] - z[thr]}{1 - F_x[thr]} - \frac{z[thr] - E[z|x < thr]}{F_x[thr]}\right\}$$

$$= \frac{p[thr]}{F_x[thr](1 - F_x[thr])}\left\{E[z|x \geq thr]F_x[thr] - z[thr] + E[z|x < thr](1 - F_x[thr])\right\}$$

⇓

$$E[z|x] = a + bF_x[x] \Longrightarrow dV/dthr = 0$$

Six datasets where $E[z|x] = a + b\,F_x[x]$

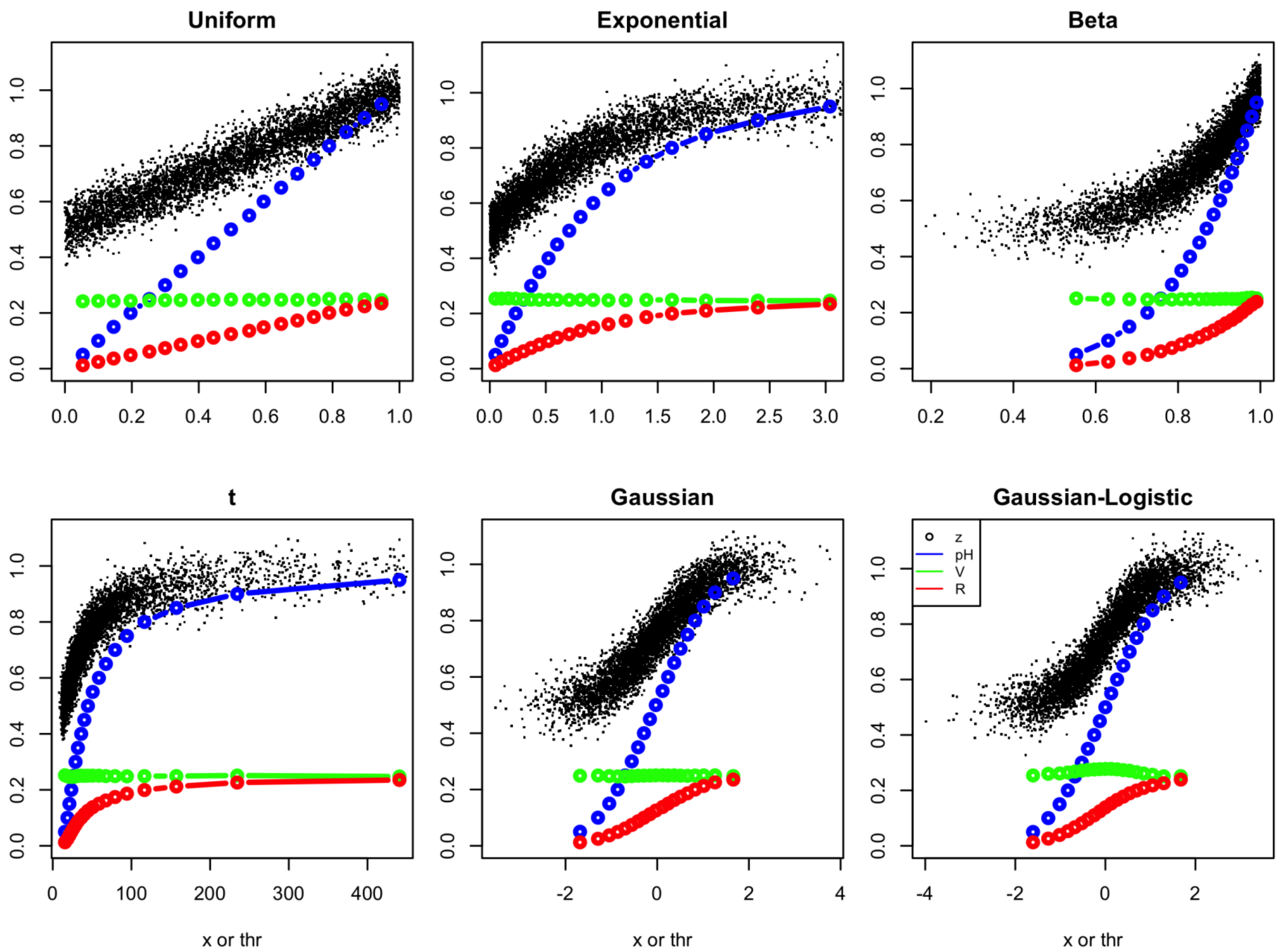
x $p[x]$ z a $b\,F_x[x]$ ε

```
x_U <- runif(n)       ; z_U <- 0.5 + x_U/2                 + rnorm(n,0,0.05)

x_E <- rexp(n)        ; z_E <- 1   - exp(-x_E)/2           + rnorm(n,0,0.05)

x_B <- rbeta(n,5,1)   ; z_B <- 0.5 + pbeta(x_B,5,1)/2      + rnorm(n,0,0.05)

x_t <- rt(n,1,30)     ; z_t <- 0.5 + pt(x_t,1,30)/2        + rnorm(n,0,0.05)

x_G <- rnorm(n)       ; z_G <- 0.5 + pnorm(x_G)/2          + rnorm(n,0,0.05)

x_L <- rnorm(n)       ; z_L <- 0.5 + 0.5/(1+exp(-2*x_L))   + rnorm(n,0,0.05)
```

We now will do **single**-threshold PRA on each of these six datasets.

We vary the location of the threshold: 5% quantile of x, 10%, …, 95%.
(So p[H] will vary accordingly.)

Single-threshold PRA on the six datasets

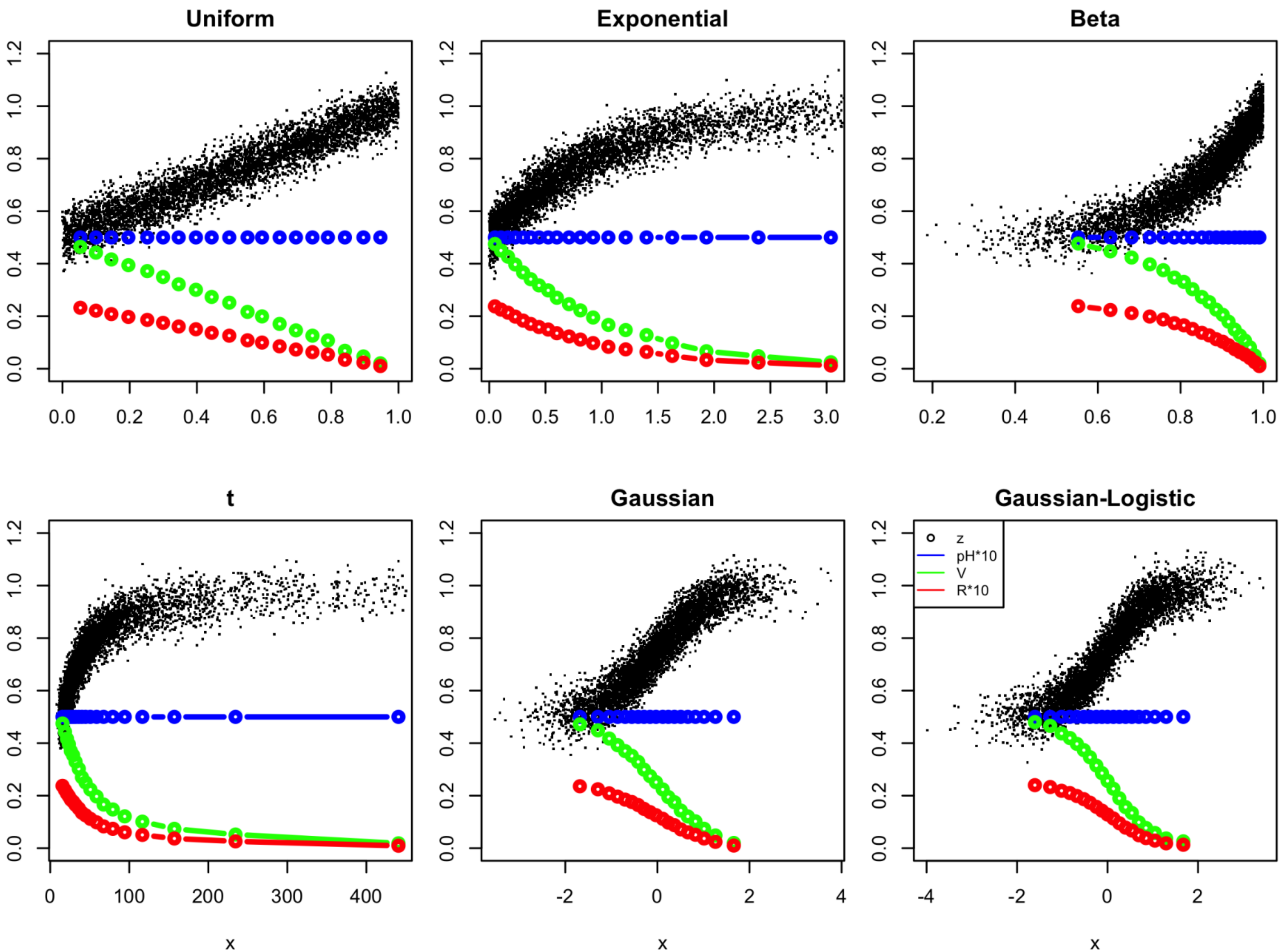


EXERCISE

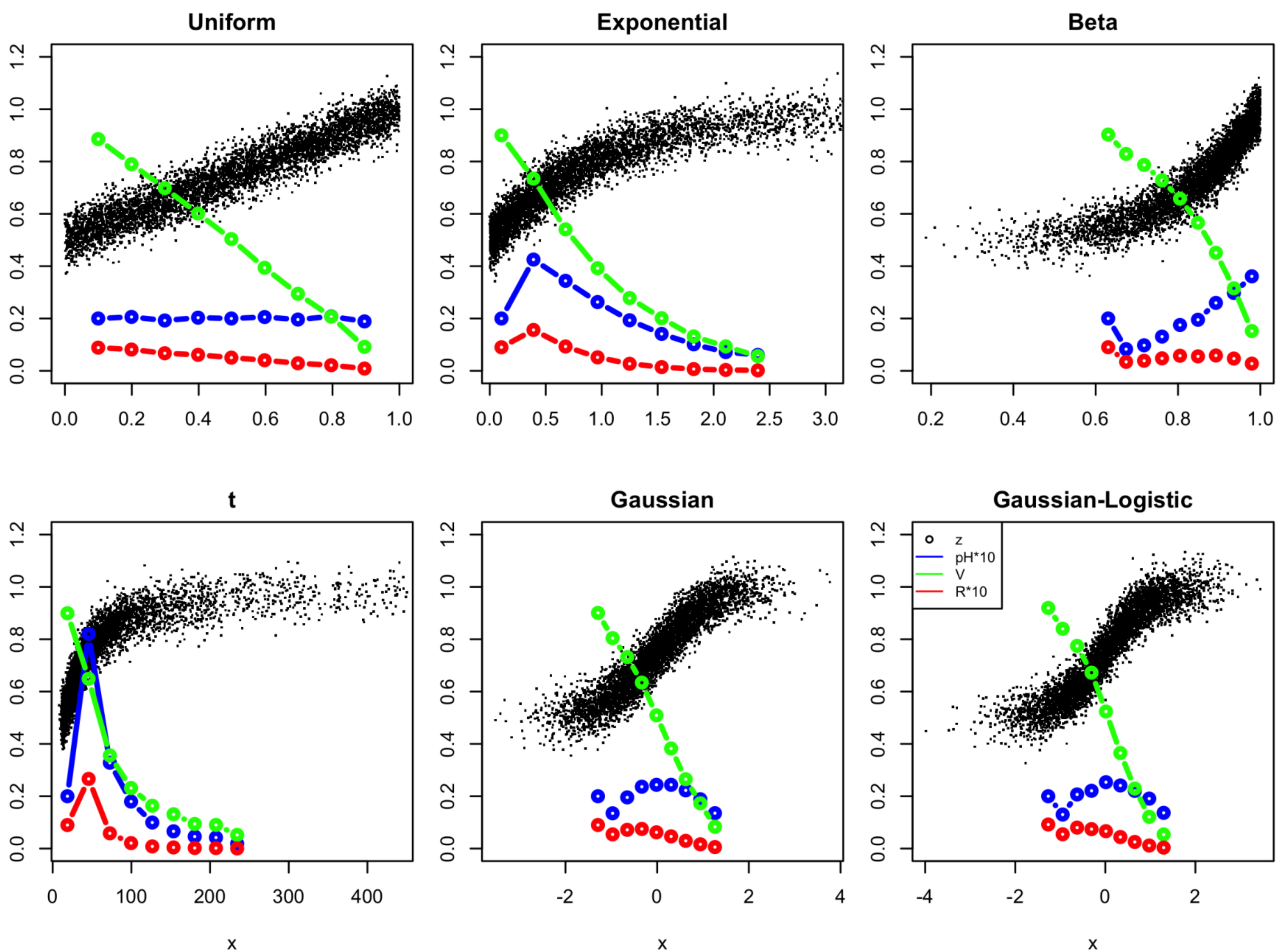
What kind of figures do you expect to see if we carry out **<u>multi</u>**-threshold PRA on these six datasets, using the same 19 threshold levels to define our x-intervals?

1. Would V still be constant, i.e. independent of the x-interval?
2. How would p[H] vary between the intervals?

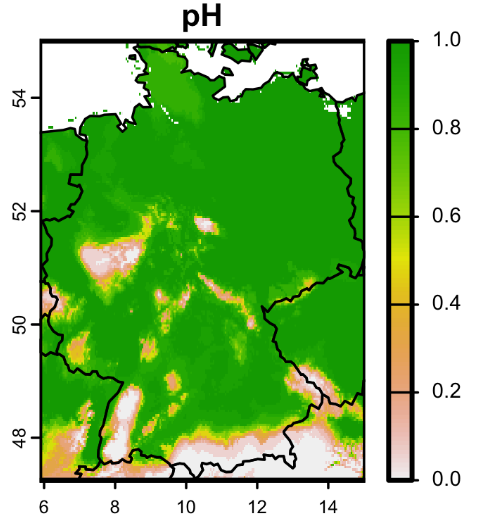
Multi-threshold PRA with equal-p[H] intervals



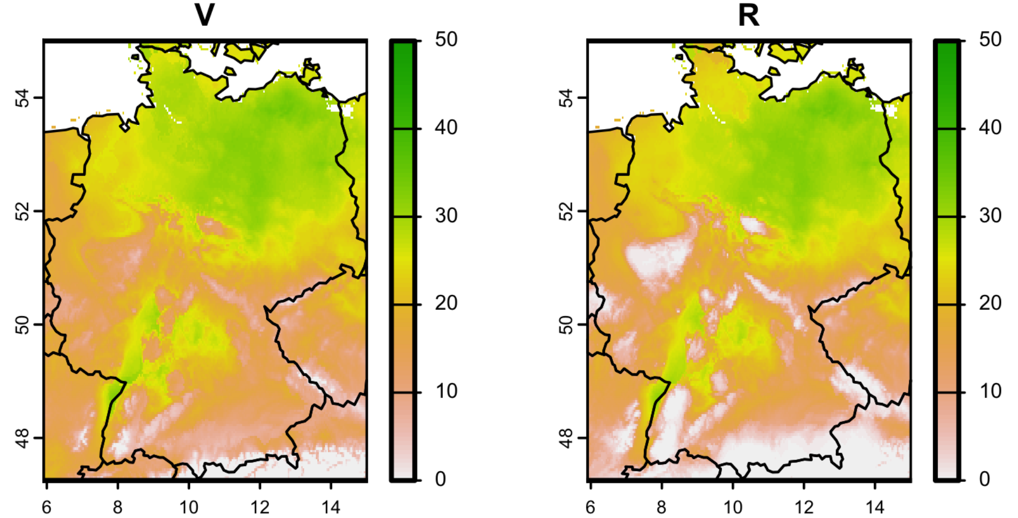
Multi-threshold PRA with equal-x-width intervals



Absolute or relative?



Absolute units ($m^3\ ha^{-1}\ y^{-1}$, kg, €, £, …)



Relative units (%)