

## Optimality Criteria in Design of Experiments: A Review

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Orthogonality and balance are known as two fundamental desirable properties in the design of experiments. They provide some kind of uniform optimality. However, in general situations, these properties cannot be met, nor do they serve particular purposes for which the experiment shall be performed. In this presentation we will give an overview of various design criteria and explain the motivation behind them: Alphabetical criteria ( $A$ ,  $c$ ,  $D$ ,  $E$ ,  $L$  as special cases of Kiefer's  $\Phi_q$ -criteria,  $MV$ , and  $R$ ) based on the risk for estimating the model parameters or parts thereof, criteria based on the risk for prediction of the mean response ( $G$ ,  $IMSE = I = Q = V$  as special cases in the Dette–O'Brien class of  $I_L$ -criteria) or individual responses, criteria for discrimination between rival models ( $T$ ), and their potential adaptation in a Bayesian setup. The abbreviations for the criteria will be demystified in the presentation. Similarities and differences between the criteria will be illustrated by simple examples of treatment-control, linear and logistic regression, and the Rasch model. Further, the entropy based approach will be mentioned.

### Further reading

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