Bayesian inference for quantiles of a log-normal variable in presence of censored observations

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The log-normality assumption is widely used to model environmental variables such as exposure concentrations. When analyzing these variables, censored observations can occurr, such as concentrations that are below the instrumental detection limit and are therefore set equal to the detection limit. More in general, we consider both right and left censoring of observations.

The availability of computational tools that allow to easy implement MCMC algorithms, makes the Bayesian approach attractive to deal with such inferential issues. However, it is known that the posterior moments of relevant quantities may not exist under the log-normal model, and quantiles are among them. As known, quantiles are crucial, for example, in estimating extreme values, assessing the exposure to pollutants in occupational health, and environmental monitoring.

We derived the existence conditions that guarantee the posterior moments of log-normal quantiles, involving the prior on the variance in the log-scale. Consequently, a prior setting coherent with these findings is proposed, choosing a generalized inverse Gaussian distribution as prior for the log-scale variance. The performances of the Bayesian estimators obtained are studied by means of a simulation study.