Comparison of the effectiveness of model selection methods in the presence of spatial covariances

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Outline

- Motivation and objectives
- Investigated designs
- Investigated model selection criteria
- Comparison of the investigated criteria
- Conclusions
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Motivation and objectives

- Spatial variability among experimental units is a typical feature in many experiments.
- Spatial variability can result from soil characteristics, microclimate etc.
- Important spatial covariances lead to different consequences in case of a true null or alternative hypothesis.
Empirical Type I error under the Null Hypotheses

<- true models ->
Empirical Type I error under the Null Hypotheses
Empirical Type I error under the Null Hypotheses

- block (fix)
- block (random)
- spatial (exp)
- spatial (sph)

<- true models ->
Empirical power under the Alternative Hypotheses

<- true models->
Motivation and objectives

- Model selection is important!
- Comparison of different analytical model selection approaches for their ability to detect a true model
Outline

- Motivation and objectives
- **Investigated designs**
- Investigated model selection criteria
- Comparison of the investigated criteria
- Conclusions
Randomized complete block design (RCB) (block fix)

varieties \( i = 1, \ldots, 20; \ r = 1, \ldots, 4 \) replications

<table>
<thead>
<tr>
<th>replication</th>
<th>1</th>
<th>2</th>
<th>3</th>
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<tbody>
<tr>
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<td>13</td>
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</tbody>
</table>

\[
y_{ij} = \mu + \alpha_i + b_l_j + e_{ij}
\]

\[
\text{var}(y) = V
\]

\[
\sigma^2_e = 11
\]

\[
bl_{1\ldots4} = 2\ldots5
\]

\[
V = \begin{bmatrix}
\sigma^2_e & 0 & \cdots & 0 & 0 \\
0 & \sigma^2_e & \cdots & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & \cdots & \sigma^2_e & 0 \\
0 & 0 & \cdots & 0 & \sigma^2_e \\
\end{bmatrix}
\]
Randomized complete block design (RCB) (block random)

varieties i =1,...,20; r=1,…4 replications

\[
y_{ij} = \mu + \alpha_i + \beta_j + e_{ij}
\]

\[
\text{var}(y) = V
\]

\[
\begin{align*}
\sigma_{\text{bl}}^2 &= 5 \\
\sigma_e^2 &= 11
\end{align*}
\]
Spatial covariance

varieties i =1,...,20; r=1,...,4 replications

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<tr>
<td>4</td>
<td>8</td>
<td>10</td>
<td>14</td>
<td>1</td>
</tr>
</tbody>
</table>

\[y_{ij} = \mu + \alpha_i + \epsilon_{ij}\]

\[\text{var}(y) = V\]

\[V = \begin{bmatrix}
\sigma^2 & \sigma_{12} & \cdots & \sigma_{1n-1} & \sigma_{1n} \\
\sigma_{21} & \sigma^2 & \cdots & \sigma_{2n-1} & \sigma_{2n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\sigma_{n-11} & \vdots & \ddots & \sigma^2 & \sigma_{i_n} \\
\sigma_{n1} & \sigma_{n2} & \cdots & \sigma_{n,n-1} & \sigma_i^2 
\end{bmatrix}\]
Spatial covariance

\[ \sigma^2 = \sigma_e^2 + \sigma_0^2 \quad (\sigma_e^2 = \text{non-spatial error variance[nugget]}) \]

\[ (\sigma_0^2 = \text{spatial variance[partial sill]}) \]

\[ \sigma_{ij} = \sigma_0^2 \cdot \exp\left( -\frac{d_{ij}}{\rho} \right) \quad (\rho = \text{range}) \quad \text{(exponential model)} \]

\[ \sigma_{ij} = \sigma_0^2 \left[ 1 - \frac{3d_{ij}}{2\rho} + \frac{d_{ij}^3}{2\rho^3} \right] (d_{ij} \leq \rho, \text{else 0})(\text{spherical model}) \]

\[ (d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}) \]
### Spatial covariance

varieties $i = 1, \ldots, 20$; $r = 1, \ldots, 4$ replications

<table>
<thead>
<tr>
<th>replication</th>
<th>variety</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>16 12 20 8 1 14 13 9 2 15</td>
</tr>
<tr>
<td></td>
<td>4 17 11 10 3 18 7 6 19 5</td>
</tr>
<tr>
<td>2</td>
<td>19 11 3 4 10 14 1 9 7 8</td>
</tr>
<tr>
<td></td>
<td>5 20 18 17 15 2 13 6 16 12</td>
</tr>
<tr>
<td>3</td>
<td>7 3 17 4 8 13 1 11 14 9</td>
</tr>
<tr>
<td></td>
<td>2 20 12 16 15 19 5 10 18 6</td>
</tr>
<tr>
<td>4</td>
<td>4 8 20 7 13 17 12 5 15 1</td>
</tr>
<tr>
<td></td>
<td>6 3 11 16 4 9 18 10 2 19</td>
</tr>
</tbody>
</table>

- $y_{ij} = \mu + \alpha_i + \epsilon_{ij}$
- $\text{var}(y) = V$
- $\sigma_0^2 = 10$
- $\rho = 20$
- $\sigma_e^2 = 1$
Outline

- Motivation and objectives
- Investigated designs
- Investigated model selection criteria
- Comparison of the investigated criteria
- Conclusions
Model selection criteria – the principle

criterion = goodness of fit + model complexity

Information Criterion of Akaike (1974):

\[ AIC = -2 \cdot \log L(\hat{\theta} | y) + 2(p_{\text{Rank}} + q) \]

\[ \hat{\theta} = (\hat{\beta}, \hat{\sigma}^2)' \]
### Investigated model selection criteria (1)
("smaller is better" – versions)

<table>
<thead>
<tr>
<th>criterion</th>
<th>ML-method block fix</th>
<th>REML-method block random</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC (Akaike, 1974)</td>
<td>-2l + 2(p_{\text{Rank}} + q)</td>
<td>-2l_R + 2q</td>
</tr>
<tr>
<td>AICC (Hurvich and Tsai, 1989)</td>
<td>-2l + 2(p_{\text{Rank}} + q)n/(n-(p_{\text{Rank}} + q)-1)</td>
<td>-2l_R + 2qn/(n-q-1)</td>
</tr>
<tr>
<td>BIC (Schwarz, 1978)</td>
<td>-2l + (p_{\text{Rank}} + q)log(n)</td>
<td>-2l_R + qlog(n*)</td>
</tr>
<tr>
<td>CAIC (Bozdogan, 1987)</td>
<td>-2l + (p_{\text{Rank}} + q)(log(n)+1)</td>
<td>-2l_R + q(log(n*)+1)</td>
</tr>
<tr>
<td>HQIC (Hannan and Quin, 1979)</td>
<td>-2l + 2(p_{\text{Rank}} + q)log(log(n))</td>
<td>-2l_R + 2qlog(log(n*))</td>
</tr>
</tbody>
</table>

**RCB(b) Spatial models:**
- $n^* = n - p_{\text{rank}}$
- $n^* = n - \text{number of blocks}$
Model selection criteria – the principle

criterion = goodness of fit + model complexity

Information Theoretic Measure of Model Complexity (ICOMP) of Bozdogan (2000):

precision and complexity of the estimates

ICOMP = -2 \cdot \log L(\hat{\theta} \mid y) + c_1 + c_2

c_1 = p_{\text{Rank}} \log(\text{Tr}(\text{Cov}_f)) - \log(\text{Det}(\text{Cov}_f))

c_2 = q \log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r))
### Investigated model selection criteria (2)
(„smaller is better“ – versions)

<table>
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<th>criterion</th>
<th>ML-method block fix</th>
<th>REML-method block random</th>
</tr>
</thead>
<tbody>
<tr>
<td>ICOMP 1 (Bozdogan, 2000)</td>
<td>-2l + c1 + c2&lt;br&gt;(c1 = p_{\text{Rank}} \log(\text{Tr}(\text{Cov}_f)) - \log(\text{Det}(\text{Cov}_f)))&lt;br&gt;(c2 = q\log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r)))</td>
<td>-2l_R + q\log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r))</td>
</tr>
<tr>
<td>ICOMP 2 (Bozdogan, 2000)</td>
<td>-2l + c&lt;br&gt;(c = (p_{\text{Rank}} + q)\log(\text{Tr}(\text{Cov}<em>{fr})) - \log(\text{Det}(\text{Cov}</em>{fr})))</td>
<td>-2l_R + q\log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r))</td>
</tr>
<tr>
<td>ICOMP (Bozdogan, 2000)</td>
<td>-2l + c1 + c2&lt;br&gt;(c1 = p_{\text{Rank}} \log(\text{Tr}(\text{Cov}_f)) - \log(\text{Det}(\text{Cov}_f)))&lt;br&gt;(c2 = q\log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r)))</td>
<td>-2l_R + q\log(\text{Tr}(\text{Cov}_r)) - \log(\text{Det}(\text{Cov}_r))</td>
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### Comparison of the criterion

**True model: RCB(block fix)**

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
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<th>CAIC</th>
<th>HQIC</th>
<th>ICOMP 1</th>
<th>ICOMP 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>RCB(fix) vs. Spat(exp)</td>
<td>0.94</td>
<td>0.83</td>
<td>0.81</td>
<td>0.75</td>
<td>0.89</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>RCB(fix) vs. Spat(sph)</td>
<td>0.89</td>
<td>0.71</td>
<td>0.67</td>
<td>0.59</td>
<td>0.81</td>
<td>0.95</td>
<td>0.96</td>
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<tr>
<td>Spat(exp) vs. Spat(sph)</td>
<td>0.18</td>
<td>0.14</td>
<td>0.14</td>
<td>0.14</td>
<td>0.15</td>
<td>0.56</td>
<td>0.55</td>
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</tbody>
</table>
Comparison of the criterion
True model: RCB(block random)

<table>
<thead>
<tr>
<th></th>
<th>AIC</th>
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<th>CAIC</th>
<th>HQIC</th>
<th>ICOMP</th>
</tr>
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<tbody>
<tr>
<td>RCB(rand) vs. Spat(exp)</td>
<td>0.96</td>
<td>0.96</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.89</td>
</tr>
<tr>
<td>RCB(rand) vs. Spat(sph)</td>
<td>0.91</td>
<td>0.91</td>
<td>0.99</td>
<td>0.99</td>
<td>0.99</td>
<td>0.93</td>
</tr>
<tr>
<td>Spat(exp) vs. Spat(sph)</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>0.67</td>
<td>0.61</td>
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Comparison of the criterion
True model: **Spatial(exp)**

<table>
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<tr>
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<tr>
<td><strong>Spat(exp) vs. RCB(fix)</strong></td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
<td>0.93</td>
<td>0.93</td>
<td>0.87</td>
<td>0.65</td>
<td>-</td>
</tr>
<tr>
<td><strong>Spat(exp) vs. RCB(rand)</strong></td>
<td>0.98</td>
<td>0.98</td>
<td>0.86</td>
<td>0.83</td>
<td>0.90</td>
<td>-</td>
<td>-</td>
<td>0.73</td>
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<tr>
<td><strong>Spat(sph) vs. RCB(fix)</strong></td>
<td>0.92</td>
<td>0.94</td>
<td>0.95</td>
<td>0.96</td>
<td>0.93</td>
<td>0.85</td>
<td>0.65</td>
<td>-</td>
</tr>
<tr>
<td><strong>Spat(sph) vs. RCB(rand)</strong></td>
<td>0.97</td>
<td>0.97</td>
<td>0.87</td>
<td>0.85</td>
<td>0.91</td>
<td>-</td>
<td>-</td>
<td>0.72</td>
</tr>
<tr>
<td><strong>Spat(exp) vs. Spat(sph)</strong></td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>0.33</td>
<td>-</td>
<td>-</td>
<td>0.40</td>
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## Comparison of the criterion

**True model: Spatial(sph)**

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<td>0.97</td>
<td>0.97</td>
<td>0.96</td>
<td>0.95</td>
<td>0.82</td>
<td>-</td>
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<td>0.99</td>
<td>0.92</td>
<td>0.90</td>
<td>0.95</td>
<td>-</td>
<td>-</td>
<td>0.89</td>
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<td>0.71</td>
<td>0.70</td>
<td>0.71</td>
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<td>-</td>
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Conclusions

- The investigated criteria have a high model selection ability in case of important block effects and spatial covariances resp.
- All in all: AIC/AICC/BIC… lead to better results as ICOMP
- ICOMP has advantages in case of RCBD(block fix)
- The application of the correct model brings important advantages for the power – model selection is necessary
Conclusions

- The results at hand do not imply that the block design approach so commonly used in experiments should be dismissed.
- They underline that by utilization of the model selection criteria a specification of the fixed and random effects has to be tested.
Thank you for your kind attention.