Comparison of the effectiveness of model selection methods in the presence of spatial covariances

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Outline

- Motivation and objectives
- Investigated designs
- Investigated model selection criteria
- Comparison of the investigated criteria
- Conclusions

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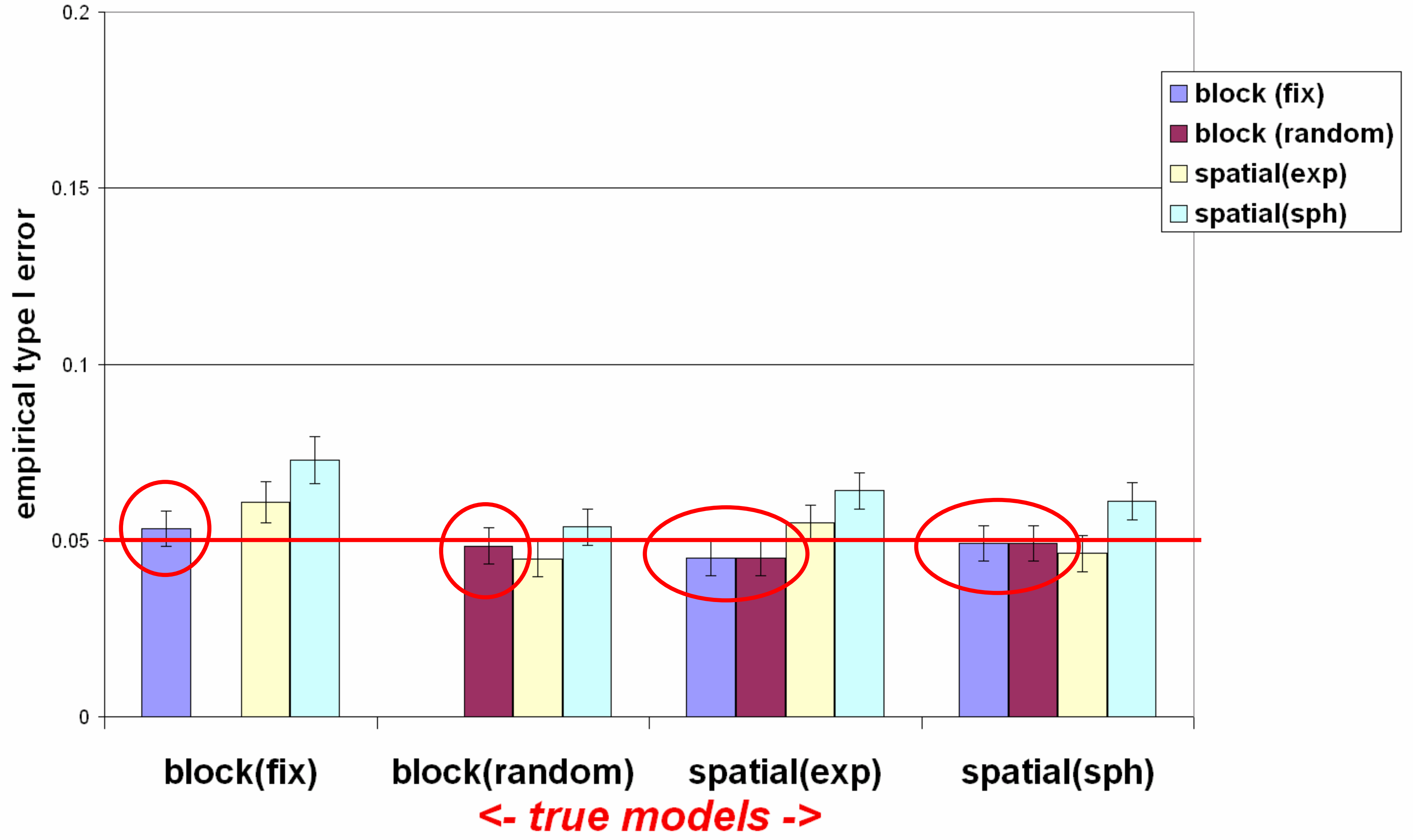
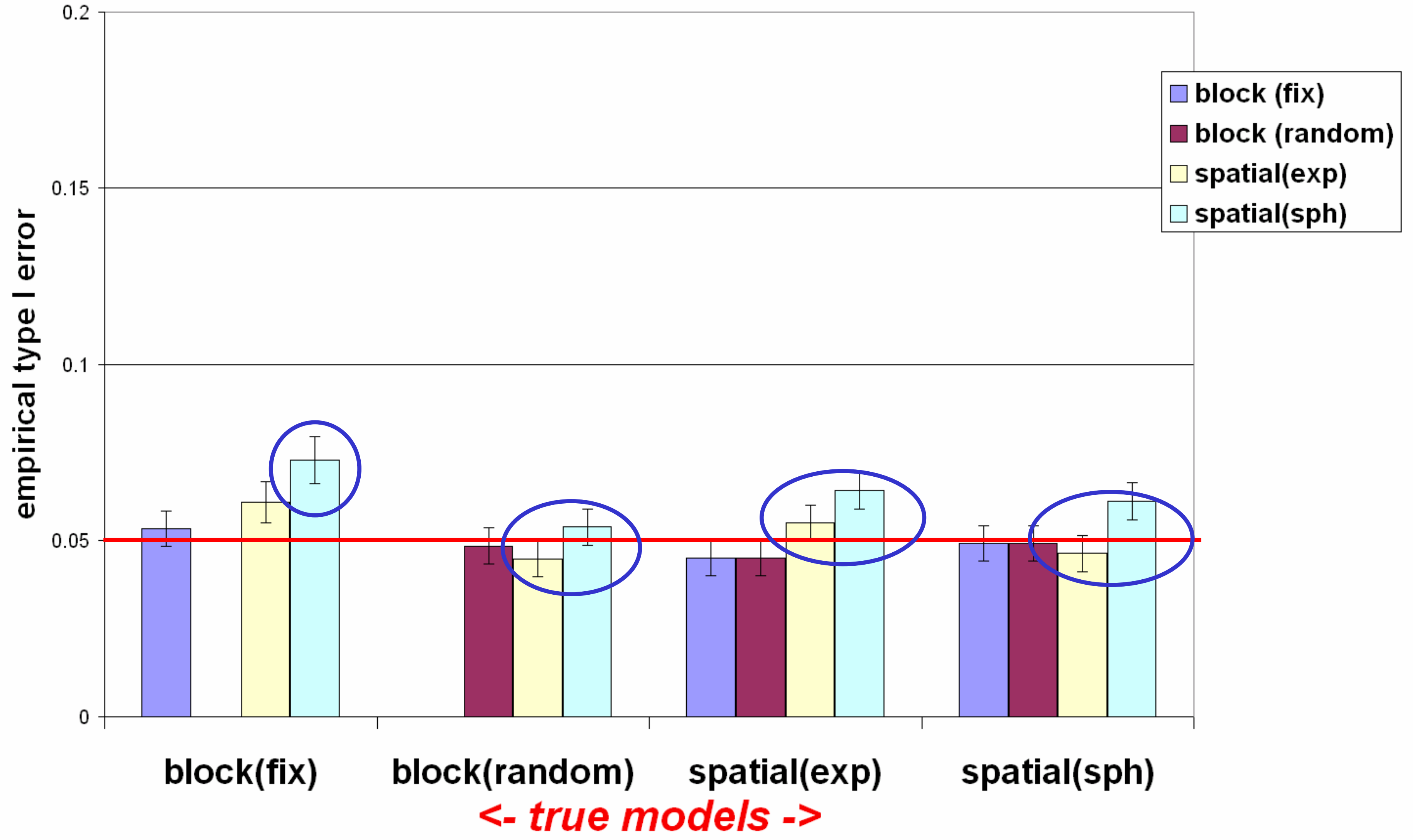
Motivation and objectives

- Spatial variability among experimental units is a typical feature in many experiments
- Spatial variability can result from soil characteristics, microclimate etc.
- Important spatial covariances lead to different consequences in case of a true null or alternative hypothesis

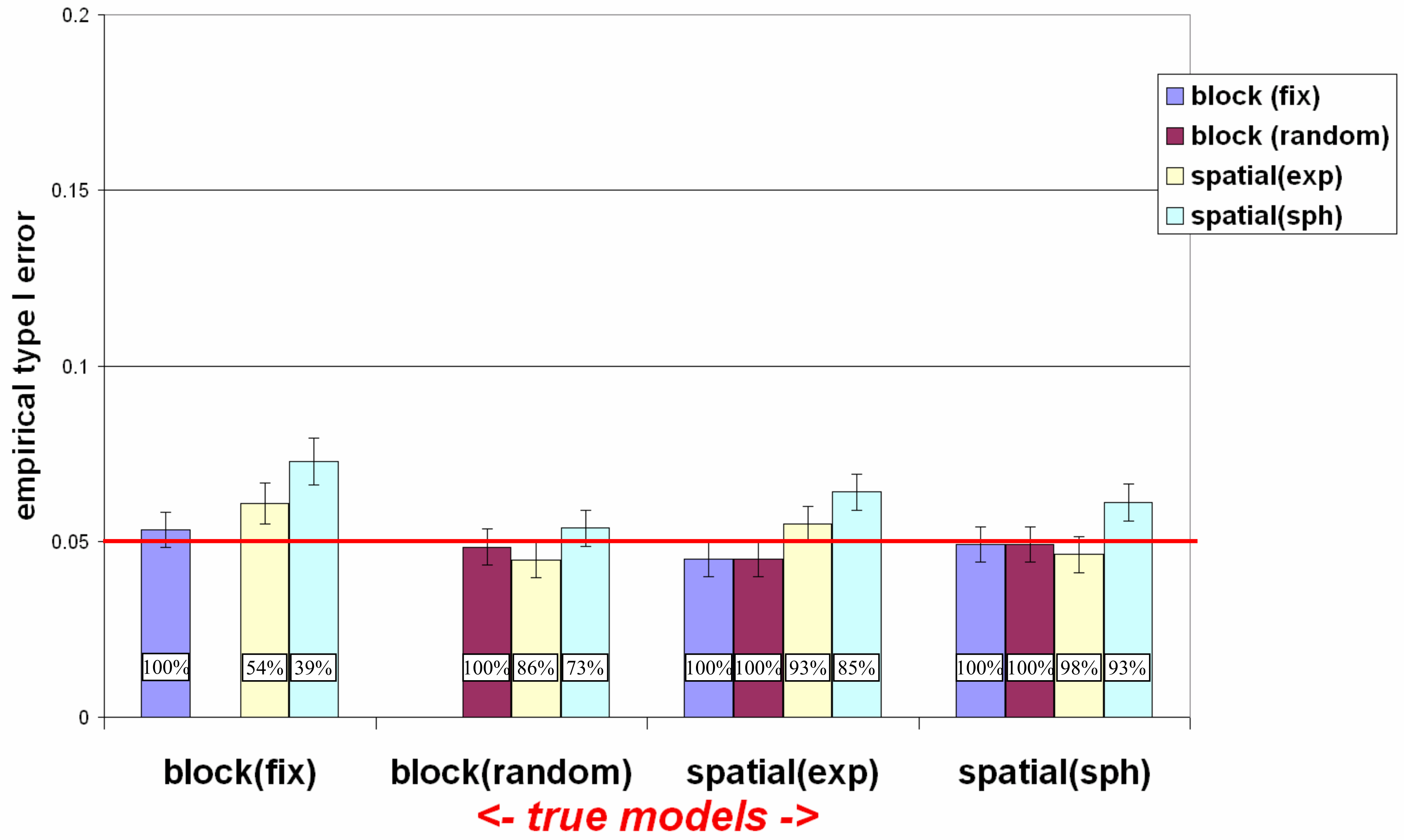
Empirical Type I error under the Null Hypotheses



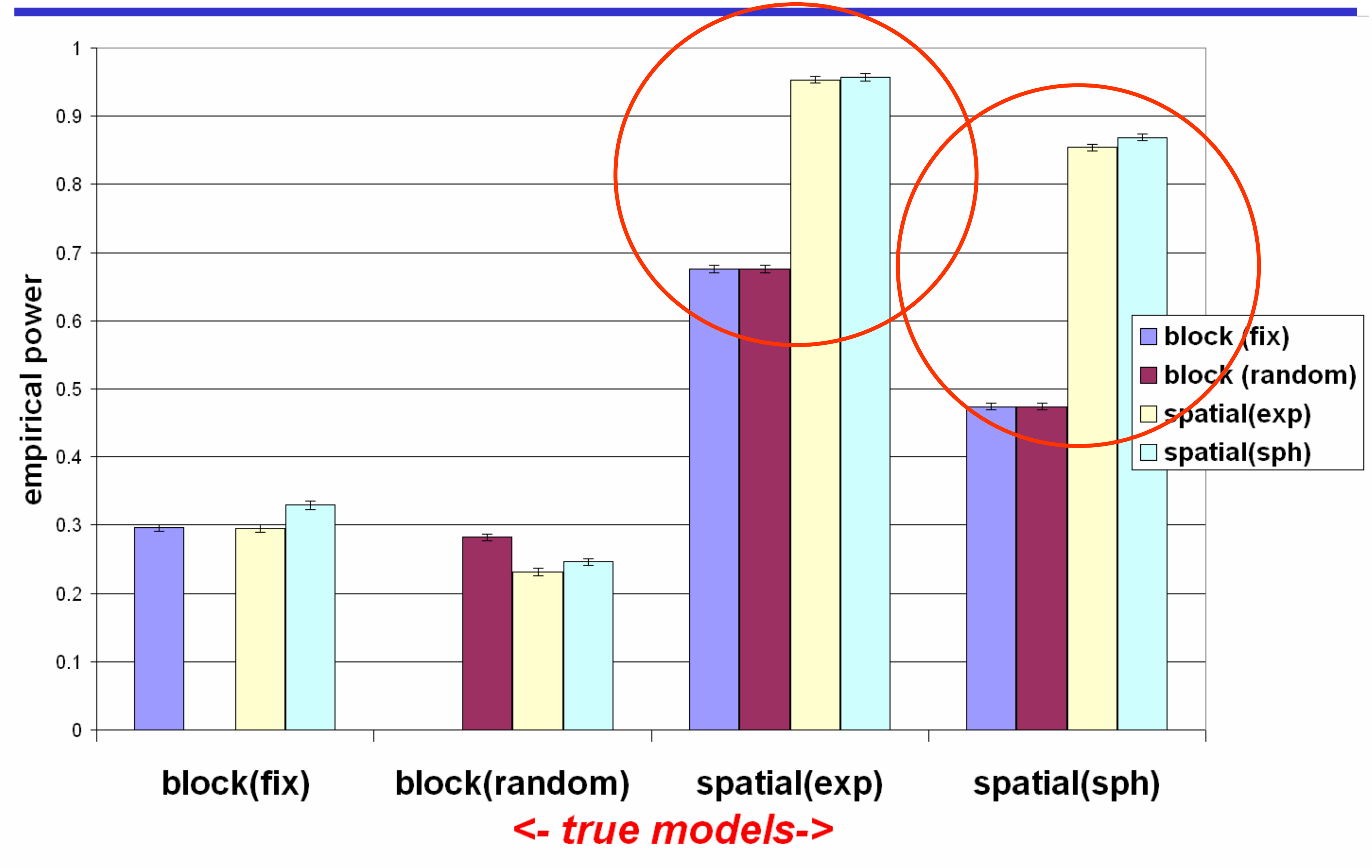
Empirical Type I error under the Null Hypotheses



Empirical Type I error under the Null Hypotheses



Empirical power under the Alternative Hypotheses



Motivation and objectives

- Model selection is important!
- Comparison of different analytical model selection approaches for their ability to detect a true model

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Randomized complete block design (RCB) (block fix)

varieties i =1,..,20; r=1,…4 replications

	variety									
replication 1	16	12	20	8	1	14	13	9	2	15
	4	17	11	10	3	18	7	6	19	5
2	19	11	3	4	10	14	1	9	7	8
	5	20	18	17	15	2	13	6	16	12
3	7	3	17	4	8	13	1	11	14	9
	2	20	12	16	15	19	5	10	18	6
4	4	8	20	7	13	17	12	5	15	1
	6	3	11	16	4	9	18	10	2	19

$$\underline{y}_{ij} = \mu + \alpha_i + bl_j + \underline{e}_{ij}$$

$$\mathrm{var}(\underline{\mathbf{y}}) = \mathbf{V}$$

$$bl_{1\ldots4} = 2\ldots5$$

$$\sigma_e^2 = 11$$

$$\mathbf{V} = \begin{bmatrix} \sigma_e^2 & 0 & \cdots & 0 & 0 \\ 0 & \sigma_e^2 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \sigma_e^2 & 0 \\ 0 & 0 & \cdots & 0 & \sigma_e^2 \end{bmatrix}$$

Randomized complete block design (RCB) (block random)

varieties i =1,..,20; r=1,…4 replications

	variety									
replication 1	**16**	**12**	**20**	**8**	**1**	**14**	**13**	**9**	**2**	**15**
	4	**17**	**11**	**10**	**3**	**18**	**7**	**6**	**19**	**5**
2	**19**	**11**	**3**	**4**	**10**	**14**	**1**	**9**	**7**	**8**
	5	**20**	**18**	**17**	**15**	**2**	**13**	**6**	**16**	**12**
3	**7**	**3**	**17**	**4**	**8**	**13**	**1**	**11**	**14**	**9**
	2	**20**	**12**	**16**	**15**	**19**	**5**	**10**	**18**	**6**
4	**4**	**8**	**20**	**7**	**13**	**17**	**12**	**5**	**15**	**1**
	6	**3**	**11**	**16**	**4**	**9**	**18**	**10**	**2**	**19**

$$\underline{y}_{ij} = \mu + \alpha_i + \underline{bl}_j + \underline{e}_{ij}$$

$$\mathrm{var}(\underline{\mathbf{y}}) = \mathbf{V}$$

$$\sigma^2_{bl} = 5$$

$$\sigma^2_e = 11$$

$$\mathbf{V} = \begin{bmatrix} \sigma^2_e + \sigma^2_{bl} & \sigma^2_{bl} & \cdots & 0 & 0 \\ \sigma_{bl} & \sigma^2_e & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \sigma^2_e + \sigma^2_{bl} & \sigma^2_{bl} \\ 0 & 0 & \cdots & \sigma^2_{bl} & \sigma^2_e + \sigma^2_{bl} \end{bmatrix}$$

Spatial covariance

varieties i =1,..,20; r=1,…4 replications

					variety					
replication 1	**16**	**12**	**20**	**8**	**1**	**14**	**13**	**9**	**2**	**15**
	4	**17**	**11**	**10**	**3**	**18**	**7**	**6**	**19**	**5**
2	**19**	**11**	**3**	**4**	**10**	**14**	**1**	**9**	**7**	**8**
	5	**20**	**18**	**17**	**15**	**2**	**13**	**6**	**16**	**12**
3	**7**	**3**	**17**	**4**	**8**	**13**	**1**	**11**	**14**	**9**
	2	**20**	**12**	**16**	**15**	**19**	**5**	**10**	**18**	**6**
4	**4**	**8**	**20**	**7**	**13**	**17**	**12**	**5**	**15**	**1**
	6	**3**	**11**	**16**	**4**	**9**	**18**	**10**	**2**	**19**

$$\underline{y}_{ij} = \mu + \alpha_i + \underline{e}_{ij}$$

$$\mathrm{var}(\underline{\mathbf{y}}) = \mathbf{V}$$

$$\mathbf{V} = \begin{bmatrix} \sigma^2 & \sigma_{12} & \cdots & \sigma_{1\,n-1} & \sigma_{1\,n} \\ \sigma_{21} & \sigma^2 & \ddots & \ddots & \sigma_{2\,n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \sigma_{n-1\,1} & \ddots & \ddots & \sigma^2 & \sigma_{i\,n} \\ \sigma_{n\,1} & \sigma_{n\,2} & \cdots & \sigma_{n\,n-1} & \sigma^2 \end{bmatrix}$$

Spatial covariance

$$\mathbf{V} = \begin{bmatrix} \sigma^2 & \sigma_{12} & \cdots & \sigma_{1\,n-1} & \sigma_{1\,n} \\ \sigma_{21} & \sigma^2 & \ddots & \ddots & \sigma_{2\,n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \sigma_{n-1\,1} & \ddots & \ddots & \sigma^2 & \sigma_{i\,n} \\ \sigma_{n\,1} & \sigma_{n\,2} & \cdots & \sigma_{n\,n-1} & \sigma^2 \end{bmatrix}$$

$$\sigma^2 = \sigma_e^2 + \sigma_0^2 \quad (\sigma_e^2 = \text{non-spatial error variance[nugget]})$$

$$(\sigma_0^2 = \text{spatial variance[partial sill]})$$

$$\sigma_{ij} = \sigma_0^2 \cdot \exp(-d_{ij}/\rho) \quad (\rho = \text{range}) \quad (\text{exponential model})$$

$$\sigma_{ij} = \sigma_0^2 [1 - \frac{3d_{ij}}{2\rho} + \frac{d_{ij}^3}{2\rho^3}] \, (d_{ij} \le \rho, \text{else } 0)(\text{spherical model})$$

$$(\mathbf{d_{ij}} = \sqrt{(\mathbf{x_i} - \mathbf{x_j})^2 + (\mathbf{y_i} - \mathbf{y_j})^2})$$

Spatial covariance

varieties i =1,..,20; r=1,…4 replications

	variety									
replication 1	**16**	**12**	**20**	**8**	**1**	**14**	**13**	**9**	**2**	**15**
	4	**17**	**11**	**10**	**3**	**18**	**7**	**6**	**19**	**5**
2	**19**	**11**	**3**	**4**	**10**	**14**	**1**	**9**	**7**	**8**
	5	**20**	**18**	**17**	**15**	**2**	**13**	**6**	**16**	**12**
3	**7**	**3**	**17**	**4**	**8**	**13**	**1**	**11**	**14**	**9**
	2	**20**	**12**	**16**	**15**	**19**	**5**	**10**	**18**	**6**
4	**4**	**8**	**20**	**7**	**13**	**17**	**12**	**5**	**15**	**1**
	6	**3**	**11**	**16**	**4**	**9**	**18**	**10**	**2**	**19**

$$\underline{y}_{ij} = \mu + \alpha_i + \underline{e}_{ij}$$

$$\mathrm{var}(\underline{\mathbf{y}}) = \mathbf{V}$$

$$\sigma_0^2 = 10$$
$$\rho = 20$$
$$\sigma_e^2 = 1$$

$$\mathbf{V} = \begin{bmatrix} \sigma^2 & \sigma_{12} & \cdots & \sigma_{1\,n-1} & \sigma_{1\,n} \\ \sigma_{21} & \sigma^2 & \ddots & \ddots & \sigma_{2\,n} \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \sigma_{n-1\,1} & \ddots & \ddots & \sigma^2 & \sigma_{i\,n} \\ \sigma_{n\,1} & \sigma_{n\,2} & \cdots & \sigma_{n\,n-1} & \sigma^2 \end{bmatrix}$$

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Model selection criteria – the prinziple

criterion = goodness of fit + model complexity

Information Criterion of Akaike(1974):

number of free parameters

ML-estimator

$$\mathrm{AIC} = -2 \cdot \log L(\hat{\theta} \mid y) + 2(p_{\mathrm{Rank}} + q)$$

$$\hat{\theta} = (\hat{\beta}, \hat{\sigma}^2)'$$

Investigated model selection criteria (1) („smaller is better“ –versions)

criterion	ML-method block fix	REML-method block random
AIC (Akaike, 1974)	$-2l + 2(p_{Rank} + q)$	$-2l_R + 2q$
AICC (Hurvich and Tsai, 1989)	$-2l + 2(p_{Rank} + q)n/(n-(p_{Rank} + q)-1)$	$-2l_R + 2qn/(n-q-1)$
BIC (Schwarz, 1978)	$-2l + (p_{Rank} + q)\log(n)$	$-2l_R + q\log(n^*)$
CAIC (Bozdogan, 1987)	$-2l + (p_{Rank} + q)(\log(n)+1)$	$-2l_R + q(\log(n^*)+1)$
HQIC (Hannan and Quin, 1979)	$-2l + 2(p_{Rank} + q)\log(\log(n))$	$-2l_R + 2q\log(\log(n^*))$

RCB(b

n*=nu ... ects

Spatial models:

$n^*=n-p_{rank}$

Model selection criteria – the prinziple

criterion = goodness of fit + model complexity

Information Theoretic Measure of Model Complexity (ICOMP) of Bozdogan (2000):

precision and complexity of the estimates

$$\mathrm{ICOMP} = -2 \cdot \log L(\hat{\theta} \mid y) + c_1 + c_2$$

$$c_1 = p_{\mathrm{Rank}} \log(\mathrm{Tr}(\mathrm{Cov}_f) - \log(\mathrm{Det}(\mathrm{Cov}_f)$$

$$c_2 = q \log(\mathrm{Tr}(\mathrm{Cov}_r) - \log(\mathrm{Det}(\mathrm{Cov}_r)$$

Investigated model selection criteria (2) („smaller is better“ –versions)

criterion	ML-method block fix	REML-method block random
ICOMP 1 (Bozdogan, 2000)	$-2l + c1 + c2$ $c1 = p_{Rank} \log(Tr(Cov_f)) - \log(Det(Cov_f))$ $c2 = q\log(Tr(Cov_r)) - \log(Det(Cov_r))$	
ICOMP 2 (Bozdogan, 2000)	$-2l + c$ $c = (p_{Rank} + q)\log(Tr(Cov_{fr})) - \log(Det(Cov_{fr}))$ $Cov_{fr} = \begin{bmatrix} Cov_f & \\ & Cov_r \end{bmatrix}$	
ICOMP (Bozdogan, 2000)		$-2l_R + q\log(Tr(Cov_r)) - \log(Det(Cov_r))$

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Comparison of the criterion
True model: RCB(block fix)

	AIC	AICC	BIC	CAIC	HQIC	ICOMP 1	ICOMP 2
RCB(fix) vs. Spat(exp)	0.94	0.83	0.81	0.75	0.89	0.94	0.95
RCB(fix) vs. Spat(sph)	0.89	0.71	0.67	0.59	0.81	0.95	0.96
Spat(exp) vs. Spat(sph)	0.18	0.14	0.14	0.14	0.15	0.56	0.55

Comparison of the criterion True model: RCB(block random)

	AIC	AICC	BIC	CAIC	HQIC	ICOMP
RCB(rand) vs. Spat(exp)	0.96	0.96	0.99	0.99	0.99	0.89
RCB(rand) vs. Spat(sph)	0.91	0.91	0.99	0.99	0.99	0.93
Spat(exp) vs. Spat(sph)	0.67	0.67	0.67	0.67	0.67	0.61

Comparison of the criterion True model: Spatial(exp)

	AIC	AICC	BIC	CAIC	HQIC	ICOMP 1	ICOMP 2	ICOMP
Spat(exp) vs. RCB(fix)	0.92	0.94	0.95	0.93	0.93	0.87	0.65	-
Spat(exp) vs. RCB(rand)	0.98	0.98	0.86	0.83	0.90	-	-	0.73
Spat(sph) vs. RCB(fix)	0.92	0.94	0.95	0.96	0.93	0.85	0.65	-
Spat(sph) vs. RCB(rand)	0.97	0.97	0.87	0.85	0.91	-	-	0.72
Spat(exp) vs. Spat(sph)	0.33	0.33	0.33	0.33	0.33	-	-	0.40

Comparison of the criterion
True model: Spatial(sph)

	AIC	AICC	BIC	CAIC	HQIC	ICOMP 1	ICOMP 2	ICOMP
Spat(sph) vs. RCB(fix)	0.96	0.97	0.97	0.98	0.96	0.95	0.85	-
Spat(sph) vs. RCB(rand)	0.99	0.99	0.93	0.91	0.95	-	-	0.87
Spat(exp) vs. RCB(fix)	0.96	0.97	0.97	0.97	0.96	0.95	0.82	-
Spat(exp) vs. RCB(rand)	0.99	0.99	0.92	0.90	0.95	-	-	0.89
Spat(sph) vs. Spat(exp)	0.71	0.71	0.71	0.70	0.71	-	-	0.68

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Conclusions

- The investigated criteria have a high model selection ability in case of important block effects and spatial covariances resp.
- All in all: AIC/AICC/BIC… lead to better results as ICOMP
- ICOMP has advantages in case of RCBD(block fix)
- The application of the correct model brings important advantages for the power – model selection is neccesary

Conclusions

- The results at hand do not imply that the block design approach so commonly used in experiments should be dismissed.
- They underline that by utilization of the model selection criteria a specification of the fixed and random effects has to be tested.

Thank you for your kind attention.