Analysis of trials with many replicates where treatments are randomised into rows and blocks

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Agricultural Field Trials – Today and Tomorrow

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Outline

- Background
- Actual trial
- Some statistical models for analyzing the results
- Simulations to evaluate the validity of the different models
- Conclusions

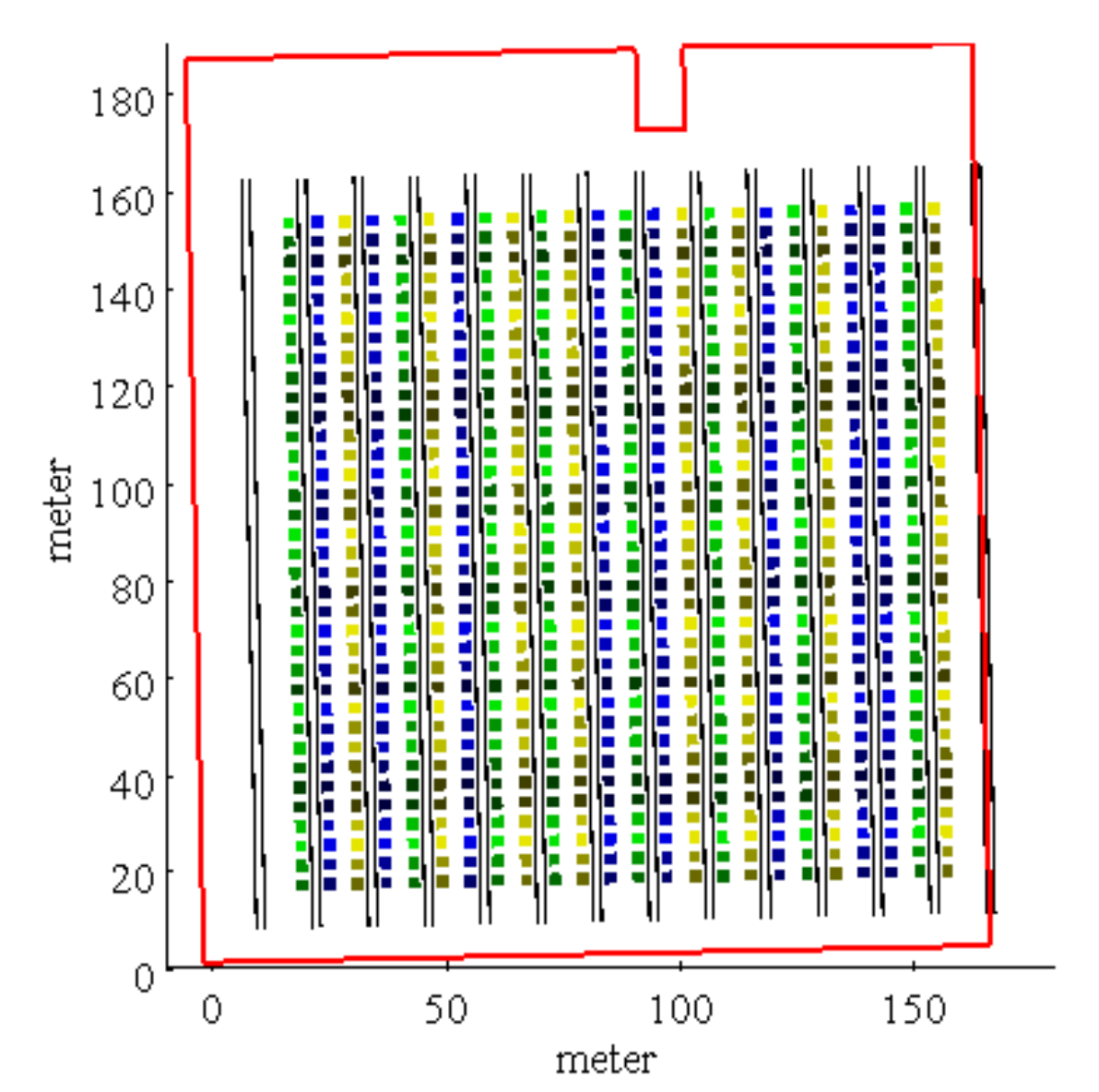
Background – robots

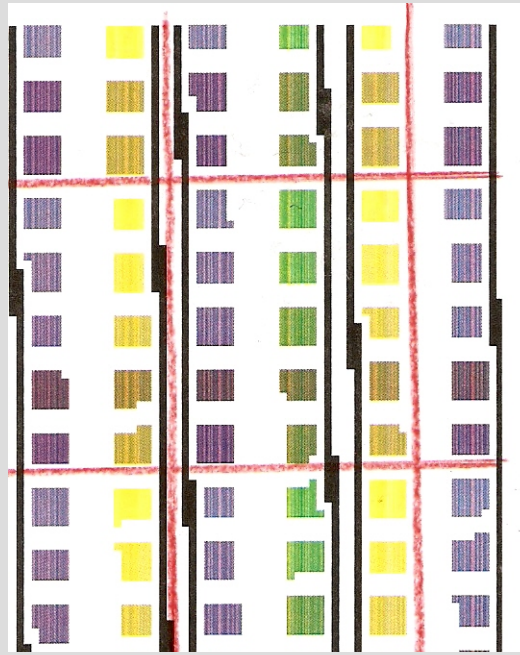
- **Robots to manage field trials**
  - Manage trials with many plots
  - Less use of man-power
- **Using spatial model in the analyses**
- **Some restrictions on randomisation may be needed**
- **Do spatial models always give valid results for such type of design?**

Actual trial

- **Spring barley, 2007**
- **Two treatment factors:**
  - Tillage at sowing: 10 cm, 5 cm and no tillage – randomised to blocks (groups) of columns
  - Plant density: 25%, 50%, 75%, 100% and 150% – randomised to blocks (groups) of rows
- **840 plots in 24 columns and 35 rows**
- **The trial covered 150 m × 150 m**
- **Harvested plot was 2.50 m × 2.34 m**
- **56 replicates**
- **Strip-plot-like design**

Actual trial





Considered methods of analysis

1. Independent observations (for comparison purpose)
2. Strip-plot-like – 6 random effects: row and column blocks, complete replicates, row and columns within blocks, residual
3. Spatial: Simple exponential
4. Spatial: Anisotropic exponential
5. Row and column effects plus simple exponential
6. Row and column effects plus anisotropic exponential
7. Exponential row effect, exponential column effect plus simple exponential residual effect

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + F_{rctd} \quad \text{Model 1}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + A_r + B_c + C_{rc} + D_{rd} + E_{ct} + F_{rctd} \quad \text{Model 2}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + G_{rctd} \quad \text{Model 3}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + H_{rctd} \quad \text{Model 4}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + D_{rd} + E_{ct} + G_{rctd} \quad \text{Model 5}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + D_{rd} + E_{ct} + H_{rctd} \quad \text{Model 6}$$

$$Y_{rctd} = \mu + \alpha_t + \beta_d + (\alpha\beta)_{tb} + K_{rd} + L_{ct} + H_{rctd} \quad \text{Model 7}$$

where

Y_{rctd} is the response recorded for the plot with tillage treatment t and plant density d in row block r and column block c

μ, α_t, β_d and $(\alpha\beta)_{tb}$ are the fixed effects

$A_r, B_c, C_{rc}, D_{rd}, E_{ct}, F_{rctd}$, are assumed to be independently normally distributed random effect

$G_{rctd}, H_{rctd}, K_{rd}$ and L_{ct} are assumed to be multivariate normally distributed random effects with covariance matrices $\Sigma_{\mathrm{G}}, \Sigma_{\mathrm{H}}, \Sigma_{\mathrm{k}}$ and Σ_l with the element i, j equal to:

$\sigma_1^2 e^{-d_{ij}/\rho}$ for Σ_{K} and Σ_L

$\sigma_1^2 e^{-d_{ij}/\rho} + \sigma_0^2 (i = j)$ for Σ_G

$\sigma_1^2 e^{-(1/\rho_1)|y_i - y_j|^{\delta_1}} e^{-(1/\rho_2)|x_i - x_j|^{\delta_2}} + \sigma_0^2 (i = j)$ for Σ_H

(y_i, x_i) and (y_j, x_j) being the coordinates of plots i and j, and $d_{ij} = \sqrt{(y_i - y_j)^2 + (x_i - x_j)^2}$

Calculated LSD-values

	Treatment main effects		Treatment combinations		
	Tillage	Density	Same Density	Same Tillage	Other
1. Indp. obs	0.083	0.107	0.186	0.186	0.186
2. Strip-plot like	0.148	0.185	0.197	0.220	0.251
3. Simple exp.	0.086	0.090	0.144	0.139	0.153
4. Aniso. exp.	0.123	0.145	0.159	0.171	0.203
5. R + C + exp.	0.151	0.187	0.181	0.209	0.246
6. R + C + aniso	0.158	0.187	0.184	0.206	0.248
7. Three exp.	0.116	0.186	0.158	0.209	0.230

LSD and Fit Statistics

	Treatment main effects		Treatment combinations			AIC
	Tillage	Density	Same Density	Same Tillage	Other	
1. Indp. obs	0.083	0.107	0.186	0.186	0.186	1238
2. Strip-plot like	0.148	0.185	0.197	0.220	0.251	1075
3. Simple exp.	0.086	0.090	0.144	0.139	0.153	948
4. Aniso. exp.	0.123	0.145	0.159	0.171	0.203	894
5. R + C + exp.	0.151	0.187	0.181	0.209	0.246	891
6. R + C + aniso	0.158	0.187	0.184	0.206	0.248	886
7. Three exp.	0.116	0.186	0.158	0.209	0.230	898

Simulations

- **Two types of spatial variations were simulated**
  1. Simple exponential (as in model 3 above): $r = 300$ m, $c_1 = 1$, $c_0 = 1$
  2. Complex exponential (sum of 3 exponential effects – as in model 7 above):
     - simple exponential effect of rows: $r = 100$ m, $c_1 = 1$
     - simple exponential effect of columns $r = 10$ m, $c_1 = 1$
     - simple exponential effect of plots: $r = 30$ m, $c_1 = 1$, $c_0 = 1$
- **Treatment randomised to rows and columns as in the actual experiment**

Simulations

- **Each type of spatial variation was simulated 1000 times and with a new randomisation of treatments for each simulation**
- **They were all analysed using 5 different models:**

1. Independent observations (for comparison purpose)
2. Strip-plot-like – 5 random effects: row and column blocks, row and columns within blocks, residual
3. Spatial: Simple exponential
5. Row and column effects plus simple exponential
7. Exponential row effect, exponential column effect plus simple exponential

Type I errors for the simple exponential random variation

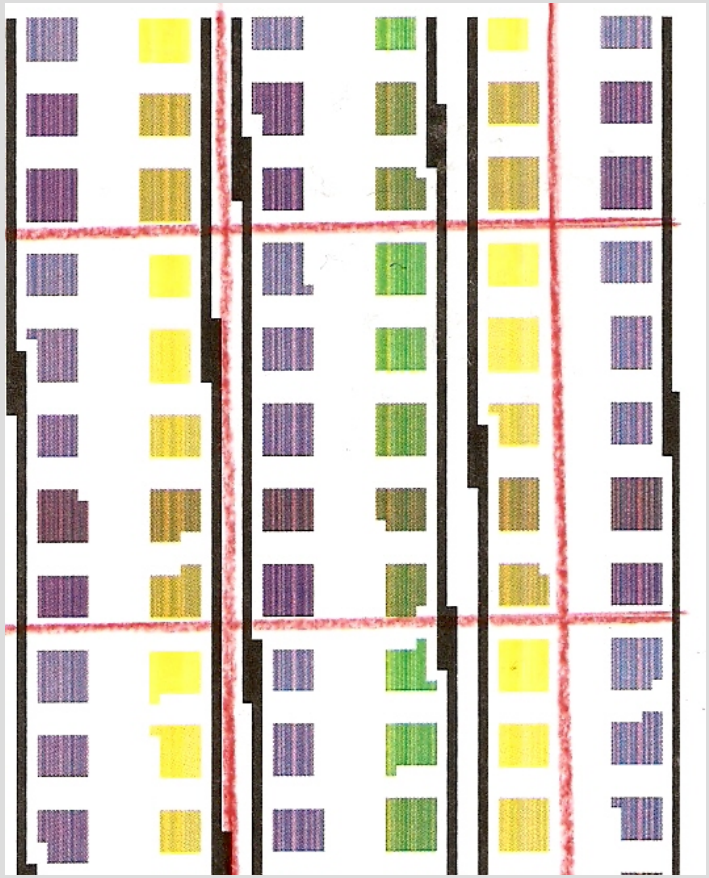
Simple exponential random variation	Treatment main effects		Treatment combinations		
	Tillage	Density	Same Density	Same Tillage	Other
1. Indp. obs	0.254	0.051	0.073	0.010	0.091
2. Strip-plot like	0.051	0.048	0.036	0.033	0.047
3. Simple exp.	0.049	0.052	0.051	0.049	0.050
5. R + C + exp.	0.034	0.033	0.041	0.045	0.039
7. Three exp.	0.036	0.032	0.040	0.039	0.038

Type I errors for the complex exponential random variation

Complex exponential random variation	Treatment main effects		Treatment combinations		
	Tillage	Density	Same Density	Same Tillage	Other
1. Indp. obs	0.467	0.080	0.146	0.019	0.176
2. Strip-plot like	0.052	0.055	0.064	0.052	0.063
3. Simple exp.	0.518	0.103	0.273	0.064	0.275
5. R + C + exp.	0.056	0.057	0.052	0.056	0.058
7. Three exp	0.047	0.053	0.050	0.053	0.052

Randomised block design

- The 15 treatments are randomised to plots within each block of 5 rows and 3 columns.
- The analyses were repeated for that design using the 1000 simulations of the complex exponential random variation



Type I errors for the complex exponential model. Randomised block design

Complex exponential random variation – randomised block	Treatment main effects		Treatment combinations		
	Tillage	Density	Same Density	Same Tillage	Other
1. Indp. obs.	0.172	0.037	0.052	0.010	0.064
x. Indp. obs.+ rep	0.047	0.046	0.049	0.049	0.049
2. Strip-plot like	0.055	0.051	0.052	0.048	0.049
3. Simple exp.	0.059	0.053	0.050	0.048	0.049
5. R + C + exp.	0.059	0.051	0.051	0.048	0.050

Conclusions - simulations

- **For the actual type of design**
  - The models that include random effects of blocks and columns had Type I errors that were close to the nominal level for both type of simulated random variation
  - The model that assumed simple exponential random variation also did well when the actual data were simulated using that model
  - The model that assumed simple exponential random variation had far too many Type I errors when the actual data were more complex

Conclusions - simulations

- **For the randomised block design**
  - The model that assumed simple exponential random variation had Type I errors that were close to the nominal level for the complex exponential random variation
  - The model with independent error did also well when the replicates (complete blocks) were included in the model
- **The simulations indicates that it is important that the structure of the random effects in the analysis reflects the randomization process**
  - If the randomisation process is not reflected there may be a risk that the Type I errors becomes far too height

LSD and fit statistics - Trial

	Treatment main effects		Treatment combinations			AIC
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Conclusions – actual trial

- **Among the assessed models, model 6 may seem to be the best because**
  - The structure of the random effects reflected the randomisation
  - This model had the smallest AIC-value
  - The strip-plot like analysis gave approximately the same LSD-values – even the AIC value was much larger