On the Behavior of Marginal and Conditional Akaike Information Criteria in Linear Mixed Models

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Akaike's information criterion (AIC) is frequently used in regression models to determine the model specification most suitable for describing a specific data generating mechanism. In particular, in linear mixed models, the AIC is often employed to differentiate between models including and excluding a specific random effect. As nonparametric regression models using penalized splines as well as spatial regression models can be recast as specific linear mixed models, this model choice decision also relates to the differentiation between linear and more general smooth or spatial models.

Two versions of the AIC have been proposed for the linear mixed model, namely the marginal AIC derived from the implied marginal model, and the conditional AIC that is build upon the conditional model formulation. We investigate theoretical properties of both, and shed light on their differences. We find that the marginal AIC is no longer an asymptotically unbiased estimator for twice the expected relative Kullback-Leibler distance, and in fact favors smaller models without random effects. This behavior is related to recent findings on the non-standard asymptotics of likelihood ratio tests for variance components, which are on the boundary of the parameter space under the null hypothesis. For the conditional AIC, we show that is important to take the uncertainty in determining the effective degrees of freedom into account when defining the conditional AIC. Ignoring the uncertainty, as is common practice, induces a bias that yields the following behavior: Whenever the random effects variance is estimated to be positive, the more complex model is preferred, regardless of the value of the estimated variance. The theoretical results are supplemented and illustrated by simulation studies, and their impact on practical data analyses is investigated in an application on childhood malnutrition in a developing country.