Convergence Diagnostics Using CODA for Optimizing the Antithetic Gibbs Sampler

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The efficiency of MCMC sampling for Markov-chain-Monte-Carlo-maximum-likelihood (MCMCML-)estimation suffers from the dependence or autocorrelation of successive iterations, which is inherent to this sampling method. In order to improve the efficiency, antithetic methods attempt to reduce this autocorrelation or even introduce negative autocorrelation. We present an application where the antithetic method is adopted to Gibbs sampling of the spatial correlation structure of Gaussian Markov random fields. In order to analyse the effect of the antithetic parameter on the properties of the sampler, we conducted simulation runs for different values of this parameter and carried out convergence diagnostics by means of CODA (Best, Cowles & Vines, 1995). This tool generates a set of statistics from each MCMC sample yielding information about the convergence to the equilibrium distribution and about the efficiency of MCMC estimation. Based on these criteria, a rule for the optimal choice of the antithetic parameter is developed. When applying this rule, the antithetic Gibbs sampler turns out to perform much better than the classical Gibbs sampler and could compete with i.i.d. sampling, which indeed is usually intractable for this kind of application.

In this application, we obtained further insight into the the properties of the convergence diagnostic techniques used by CODA. Concerning the criteria for assessing the efficiency, i.e. the variance of an ergodic mean, we observed a strong difference between the results of the batch means and the periodogram estimator on the one hand and the Rafterty & Lewis’ dependence factor on the other hand, which might be due to the transformation to a binary variable used by the latter. The first two criteria produce very similar results, but the batch means estimates of the MCMC variance tend to be slightly higher than the periodogram estimates. We also found differences between the criteria for diagnosing the convergence towards the equilibrium distribution, in particular between Geweke’s Z-scores and Heidelberger & Welch’s Cramer von Mises statistics on the one hand and the Rafterty & Lewis’ burn in length on the other hand. In contrast to Cowles & Carlin (1994) we observed a strong correspondence of the Z-score and the Cramer von Mises statistics.